Head Tracking and IMUs

CS 6334 Virtual Reality

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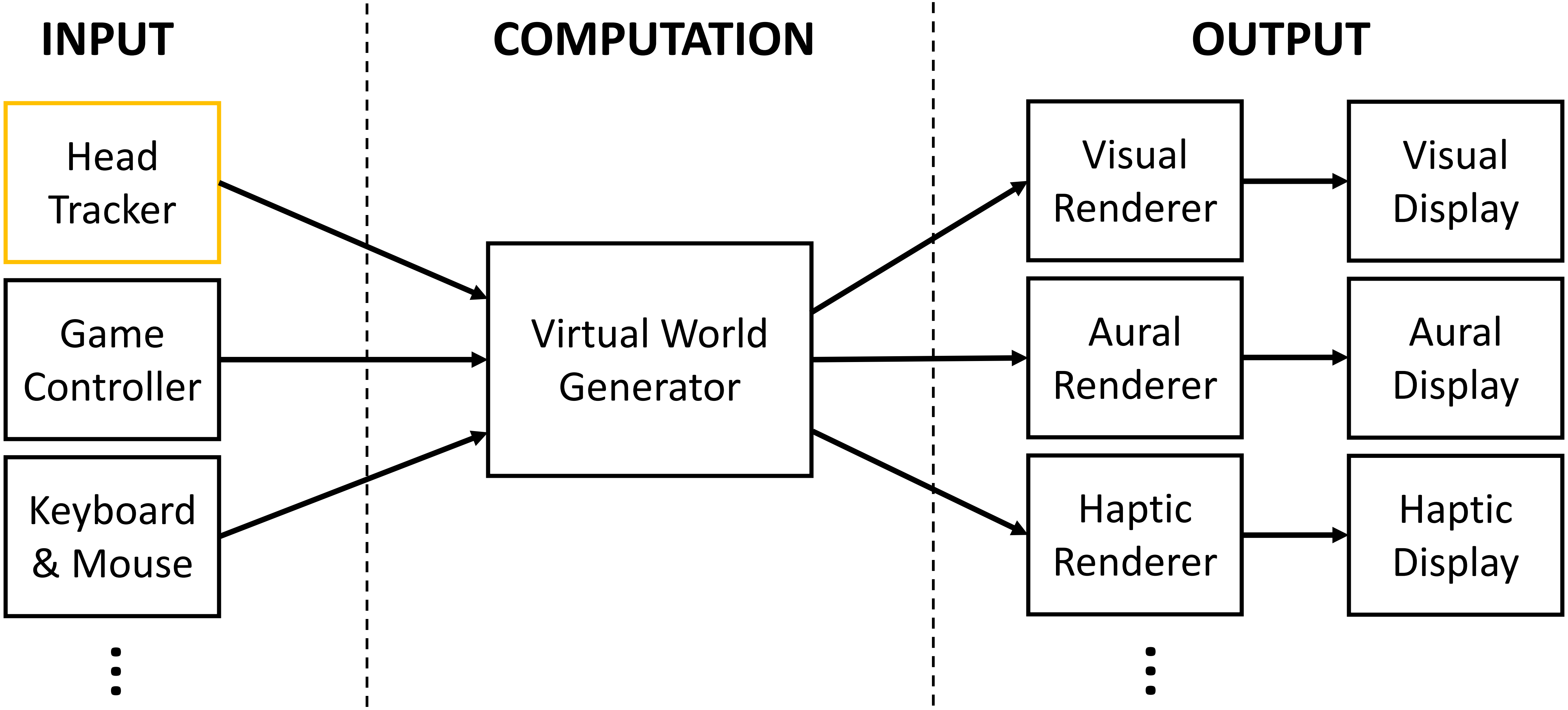
A lot of slides of course lectures borrowed from Professor Yu Xiang’s VR class

Review of VR Systems



INPUT

COMPUTATION

OUTPUT

Head Tracker

Game Controller

Keyboard & Mouse

⋮

Virtual World Generator

Visual Renderer

Visual Display

Aural Renderer

Aural Display

Haptic Renderer

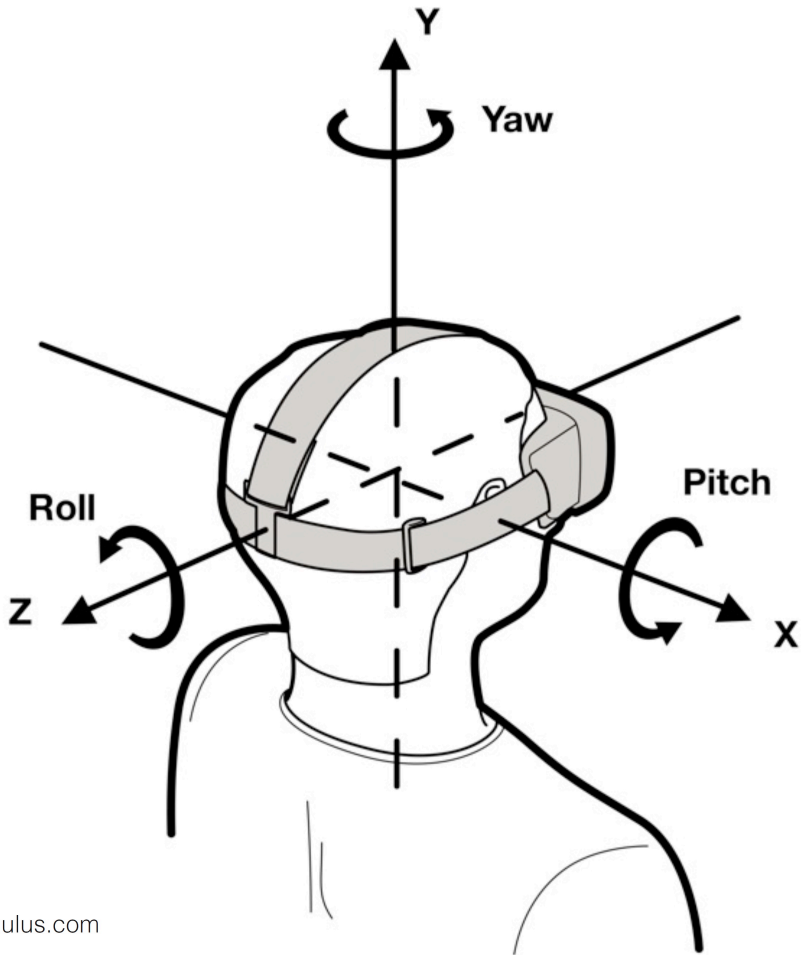
Haptic Display

⋮

Tracking in VR

- Tracking the user's sense organs
  - E.g., Head and eye
  - Render stimulus accordingly
- Tracking user's other body parts
  - E.g., human body and hands
  - Locomotion and manipulation
- Tracking the rest of the environment
  - Augmented reality
  - Obstacle avoidance in the real world

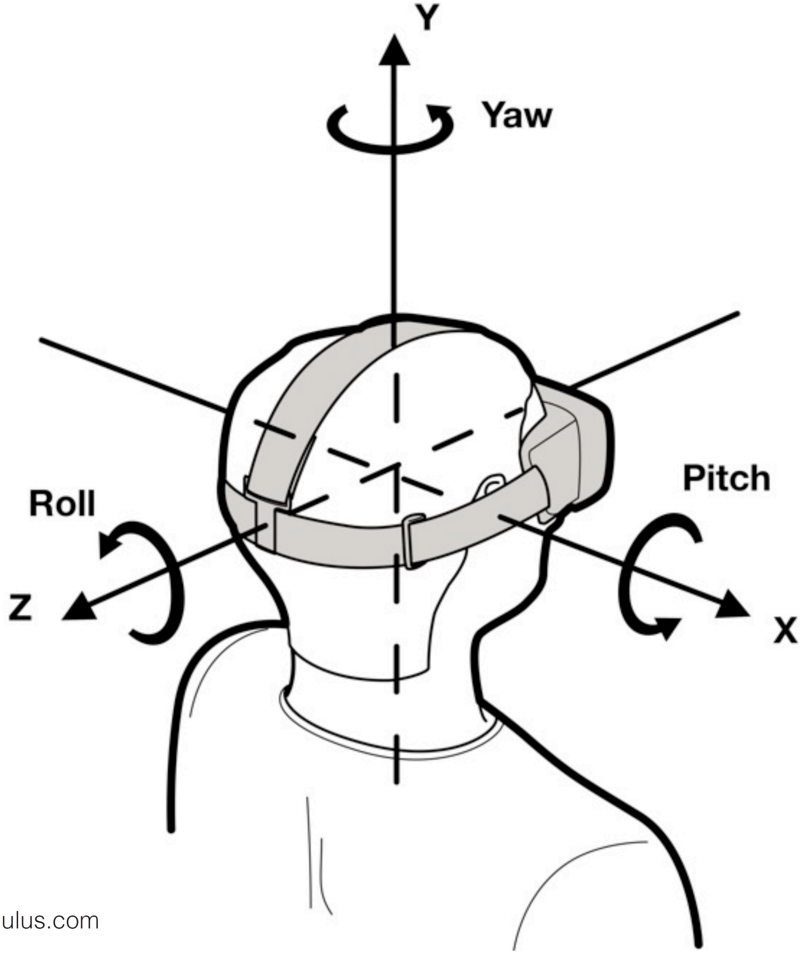
Head Tracking



oculus.com

- Track orientation of the head
- Orientation is the rotation of the device w.r.t. world or inertial frame
- Euler angle representation: yaw, pitch, roll

Head Tracking



oculus.com

- Determine the viewpoint of the user
- In visual rendering

vertex in clip space ↓ vertex ↓

$$v_{clip} = M_{proj} \cdot M_{view} \cdot M_{model} \cdot v$$

↑ projection matrix ↑ view matrix ↑ model matrix

rotation ↓ translation ↓

$$M_{view} = R \cdot T\left(-eye\right)$$

roll pitch yaw

$$R = R_z\left(-\theta_z\right) \cdot R_x\left(-\theta_x\right) \cdot R_y\left(-\theta_y\right)$$

Inertial Measurement Unit (IMU)

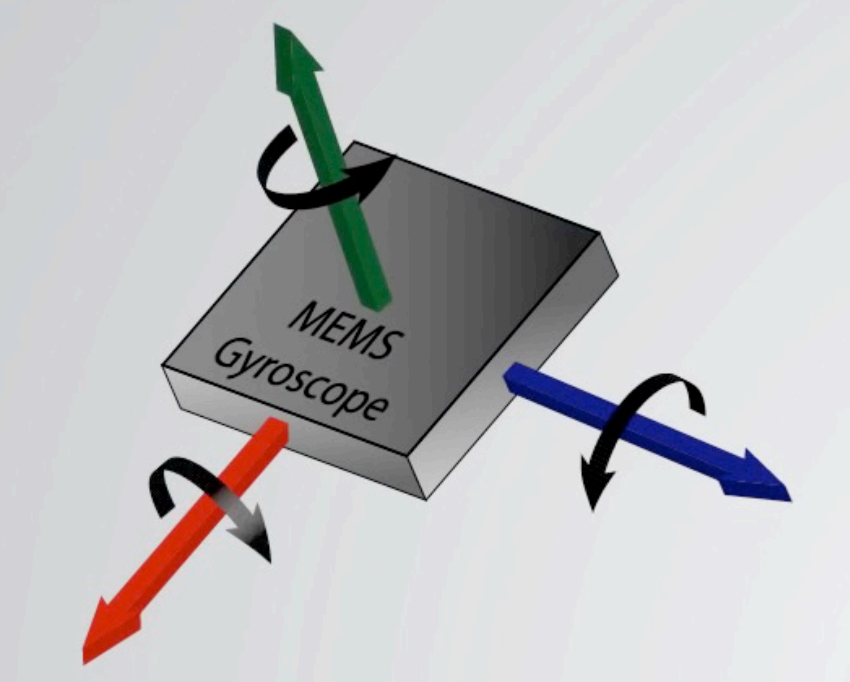
- Gyroscope measures angular velocity $\tilde{\omega}$ in degrees/second
- Accelerometer measures linear acceleration $\tilde{a}$ in m/s^2
- Magnetometer measures magnetic field strength $\tilde{m}$ in uT (micro Tesla) or Gauss, 1 Gauss = 100 uT

All measurements taken in sensory/body coordinates

Gyroscopes

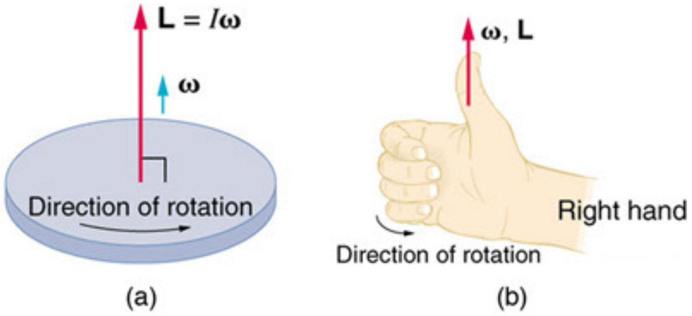
- Measure angular velocity

Microelectromechanical systems (MEMS)

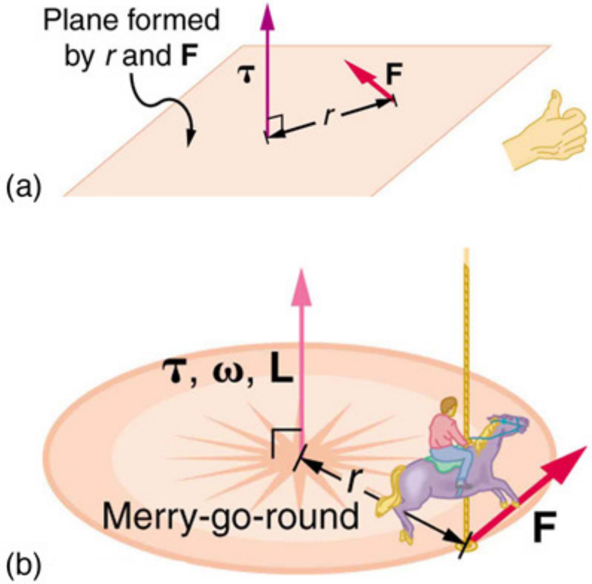


https://robotacademy.net.au/lesson/how-gyroscopes-work/

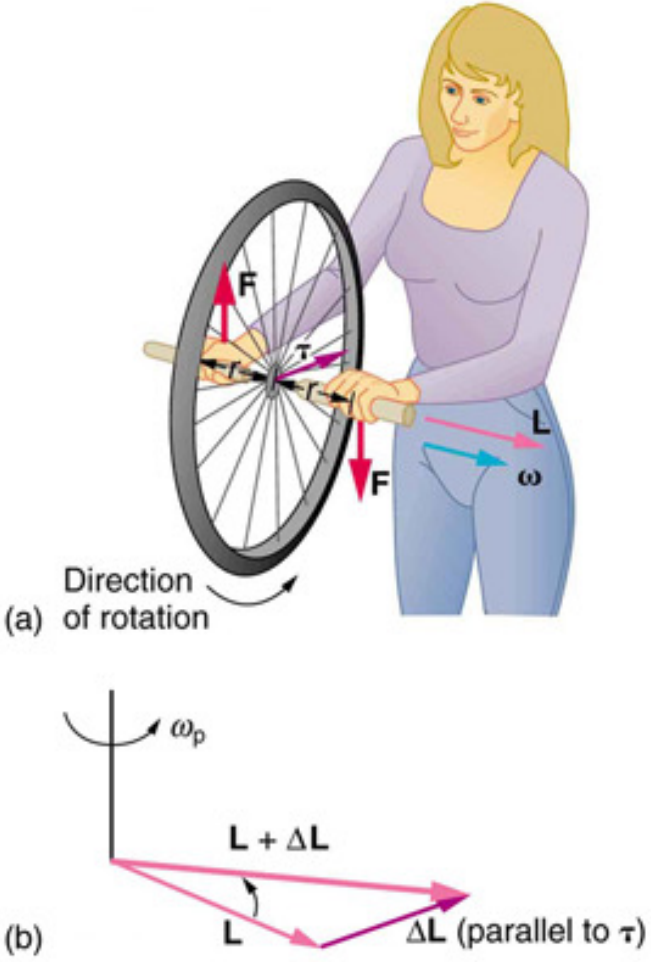
Gyroscopic Effects: Vector Aspects of Angular Momentum



Angular velocity
Angular momentum



$$\text{net } \tau = \frac{\Delta L}{\Delta t}$$



https://courses.lumenlearning.com/physics/chapter/10-7-gyroscopic-effects-vector-aspects-of-angular-momentum/
https://www.youtube.com/watch?v=8H98BgRzpOM

Gyroscopes

- Gyro model $$\tilde{\omega} = \omega + b + \eta \qquad \eta \sim N\left(0, \sigma_{gyro}^{2}\right)$$

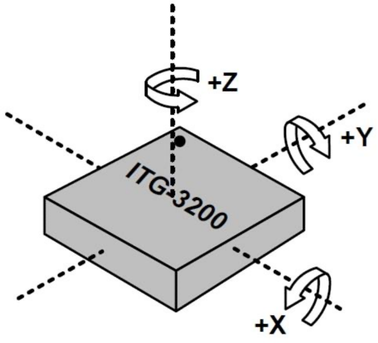
measured angular velocity

true angular velocity

bias

additive, zero-mean Gaussian noise

3DOF: 3-axis gyro that measures 3 orthogonal axes



Gyroscopes

- From gyro measurement to orientation
  - Taylor expansion

Time step

$$\theta(t+\Delta t) \approx \theta(t) + \frac{\partial}{\partial t}\theta(t)\Delta t + \varepsilon, \varepsilon \sim O(\Delta t^2)$$

Angle at current time step

Angle at previous time step

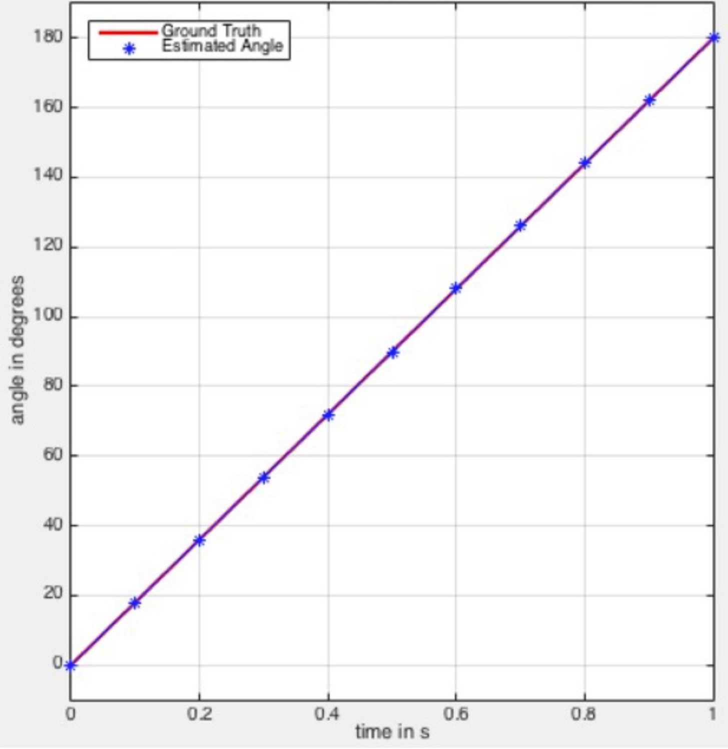
Gyro measurement (angular velocity)

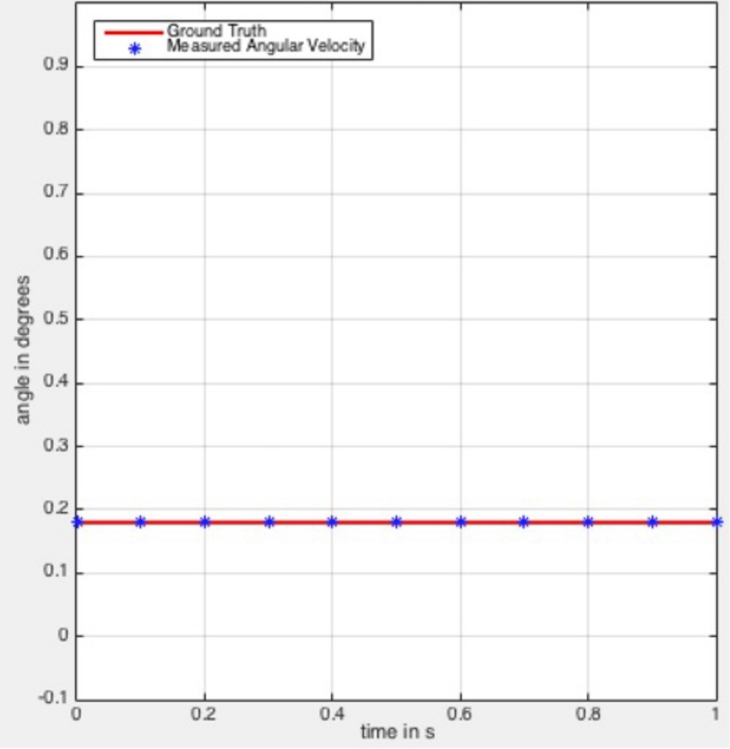
Approximation error

$$\widetilde{\omega} = \omega + b + \eta$$

Gyro Integration

- Linear motion, no noise, no bias $\theta(t+\Delta t) \approx \theta(t) + \frac{\partial}{\partial t}\theta(t)\Delta t + \varepsilon, \varepsilon \sim O(\Delta t^2)$


Orientation


Gyro Measurements: angular velocity

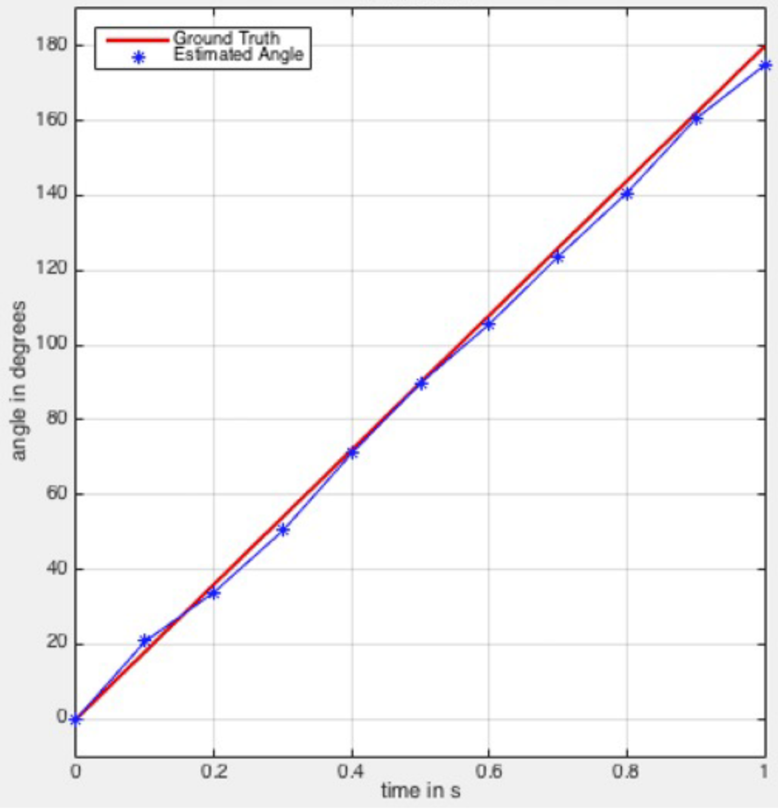
$$\tilde{\omega} = \omega + b + \eta$$

Gyro Integration

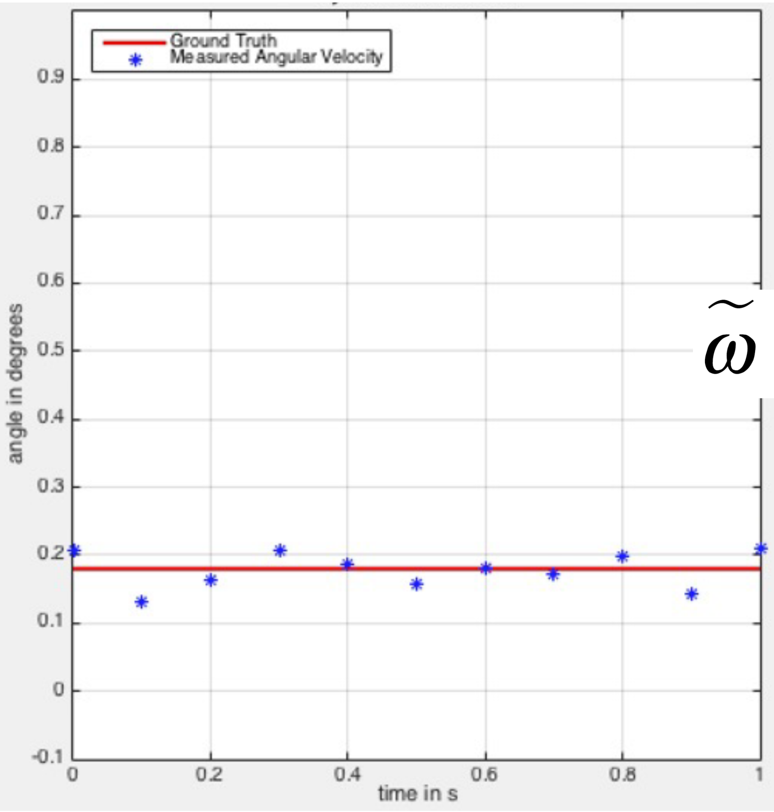
- Linear motion, noise, no bias

$$\theta(t+\Delta t) \approx \theta(t) + \frac{\partial}{\partial t}\theta(t)\Delta t + \varepsilon, \varepsilon \sim O(\Delta t^2)$$

Orientation



Gyro Measurements: angular velocity



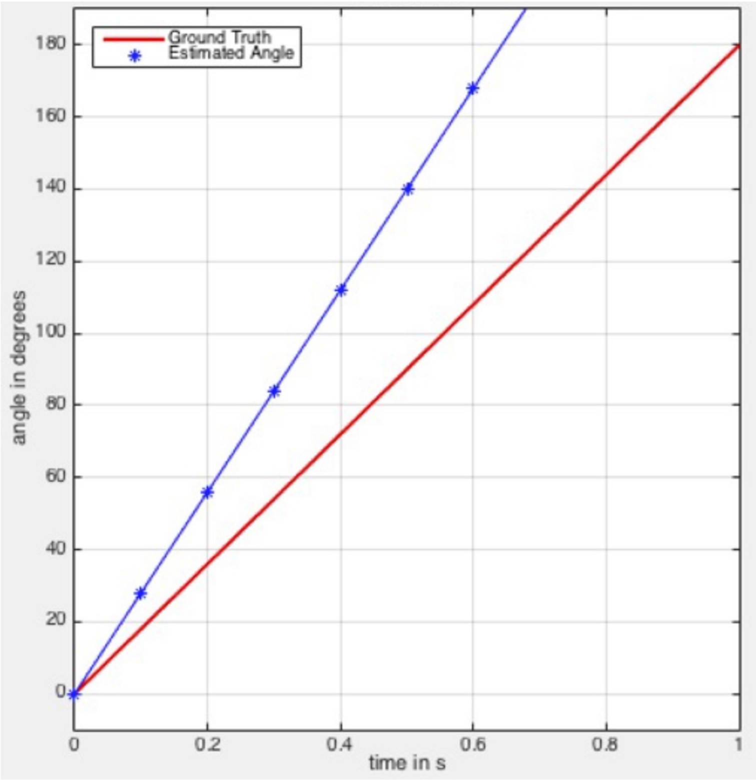
$$\tilde{\omega} = \omega + b + \eta$$

Gyro Integration

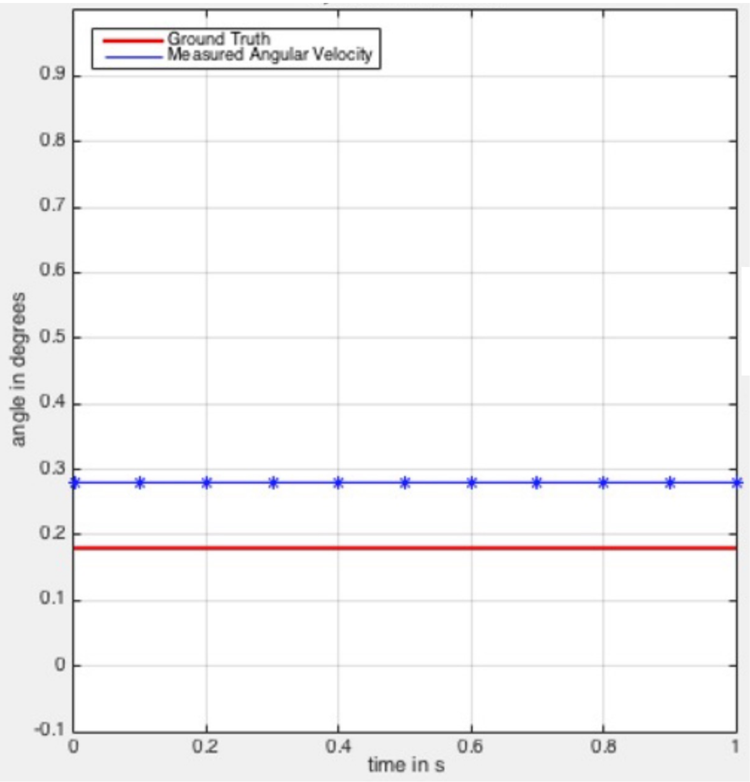
- Linear motion, no noise, bias

$$\theta(t+\Delta t) \approx \theta(t) + \frac{\partial}{\partial t}\theta(t)\Delta t + \varepsilon, \varepsilon \sim O(\Delta t^2)$$

Orientation



Gyro Measurements: angular velocity



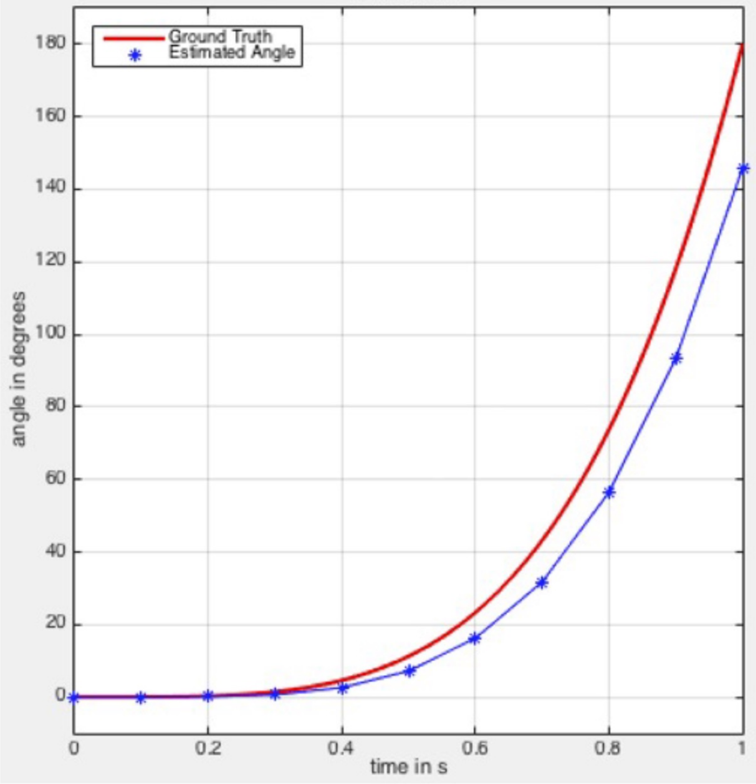
$$\widetilde{\omega} = \omega + b + \eta$$

Gyro Integration

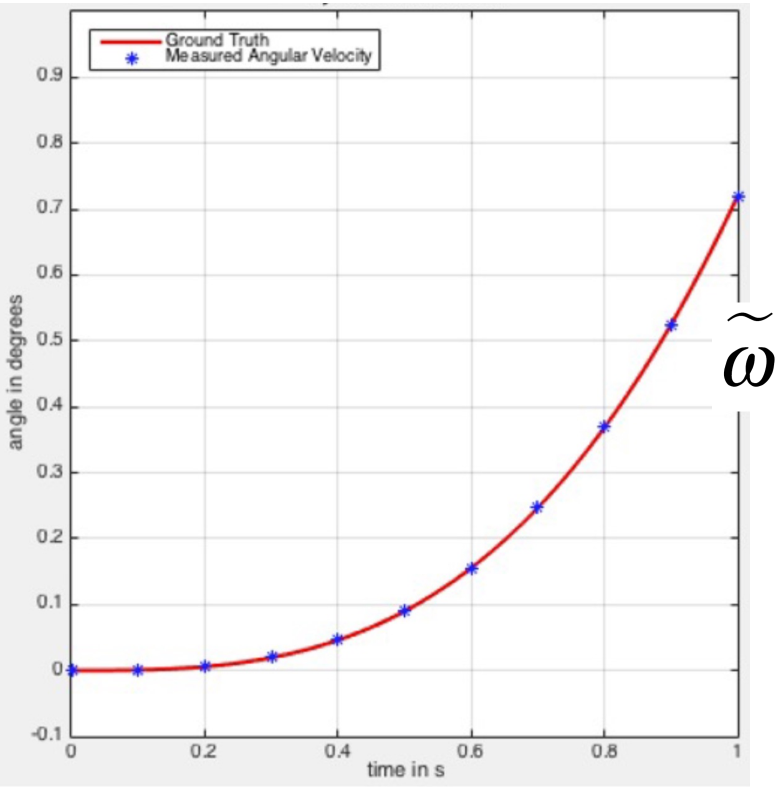
- Nonlinear motion, no noise, no bias $\theta(t+\Delta t) \approx \theta(t) + \frac{\partial}{\partial t}\theta(t)\Delta t + \varepsilon, \varepsilon \sim O(\Delta t^2)$

Due to approximation error in Taylor expansion for nonlinear motion

Orientation



Gyro Measurements: angular velocity



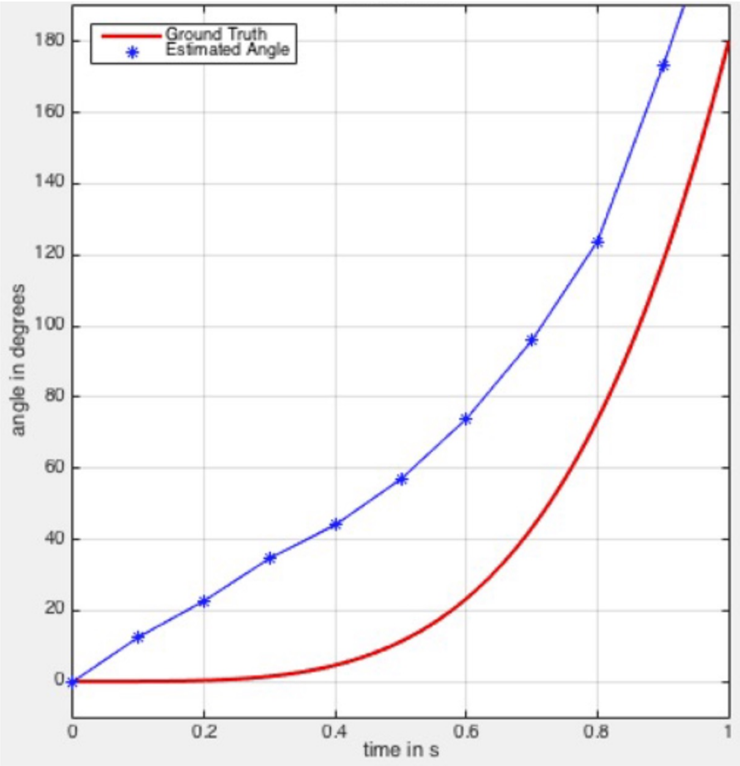
$$\widetilde{\omega} = \omega + b + \eta$$

Gyro Integration

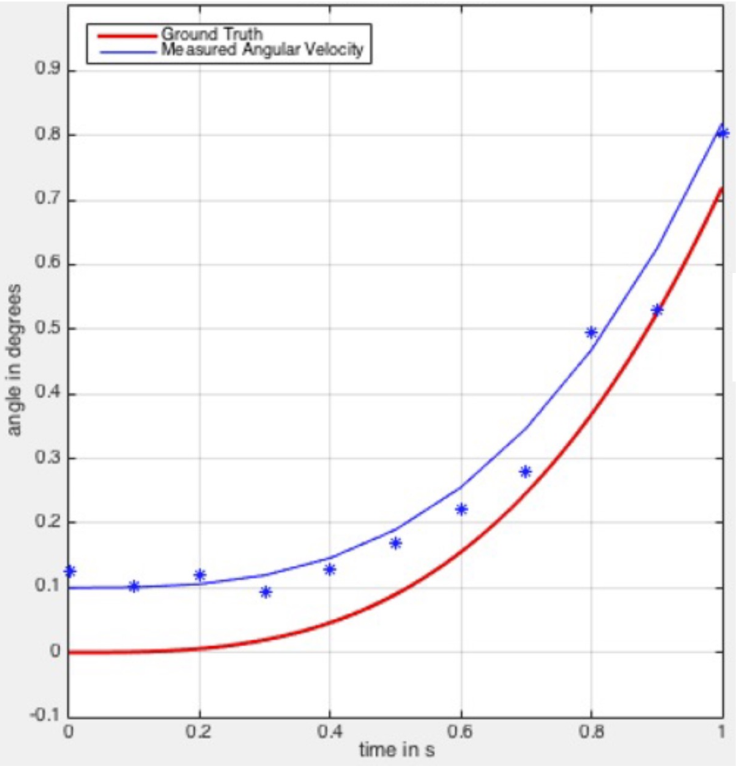
- Nonlinear motion, noise, bias

$$\theta(t+\Delta t) \approx \theta(t) + \frac{\partial}{\partial t}\theta(t)\Delta t + \varepsilon, \varepsilon \sim O(\Delta t^2)$$

Orientation



Gyro Measurements: angular velocity



$$\tilde{\omega} = \omega + b + \eta$$

Gyro Integration

- Works well for linear motion, no noise, no bias (unrealistic)

- Integration drift
    - Errors in measured angular velocity result in errors in orientation
    - Errors accumulate in time

- Gyro integration is accurate in short time, but not reliable in long term due to drift

- Bias/noise variance can be estimated, other sensor measurements can be used to correct drift, e.g., vision, accelerometer

Accelerometers

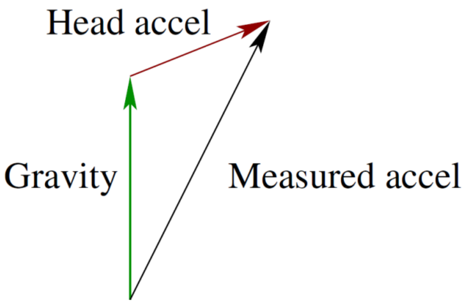
- Measure linear acceleration

$$\tilde{a} = a^{(g)} + a^{(l)} + \eta, \quad \eta \sim N\left(0, \sigma_{acc}^{2}\right)$$

Gravity acceleration (pointing up)

external acceleration

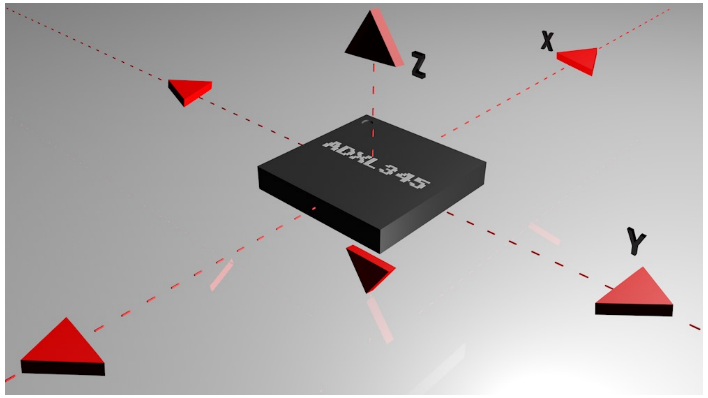
additive, zero-mean Gaussian noise



Think about the force of the table pushing the device upwards

Accelerometers

- Pros
  - Points up on average with magnitude of 9.81 m/s^2 = 1g
  - Accurate in long term because there is no drift



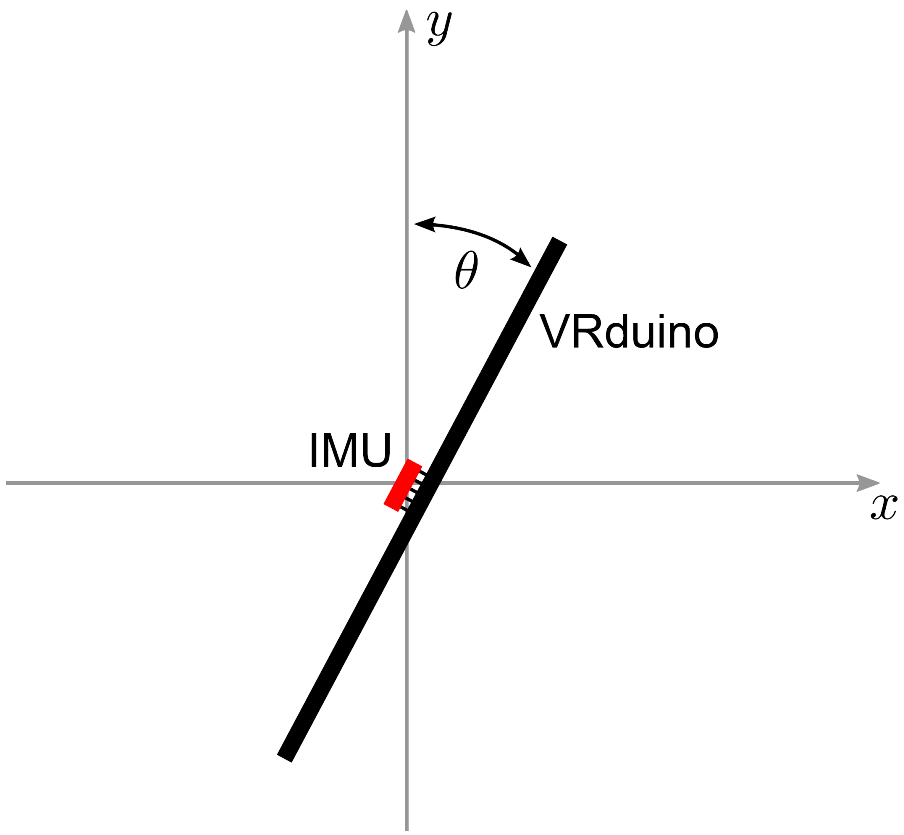
- Cons
  - Noisy measurements
  - Unreliable in short run due to motion and noise
- Complementary to gyro measurements
- Fusing gyro and accelerometer data: 6DOF sensor fusion

Orientation Tracking Example

- Track angle θ in 2D space

- Sensors
    - 1 gyro
    - 2 accelerometers

- Goal: understand sensor fusion



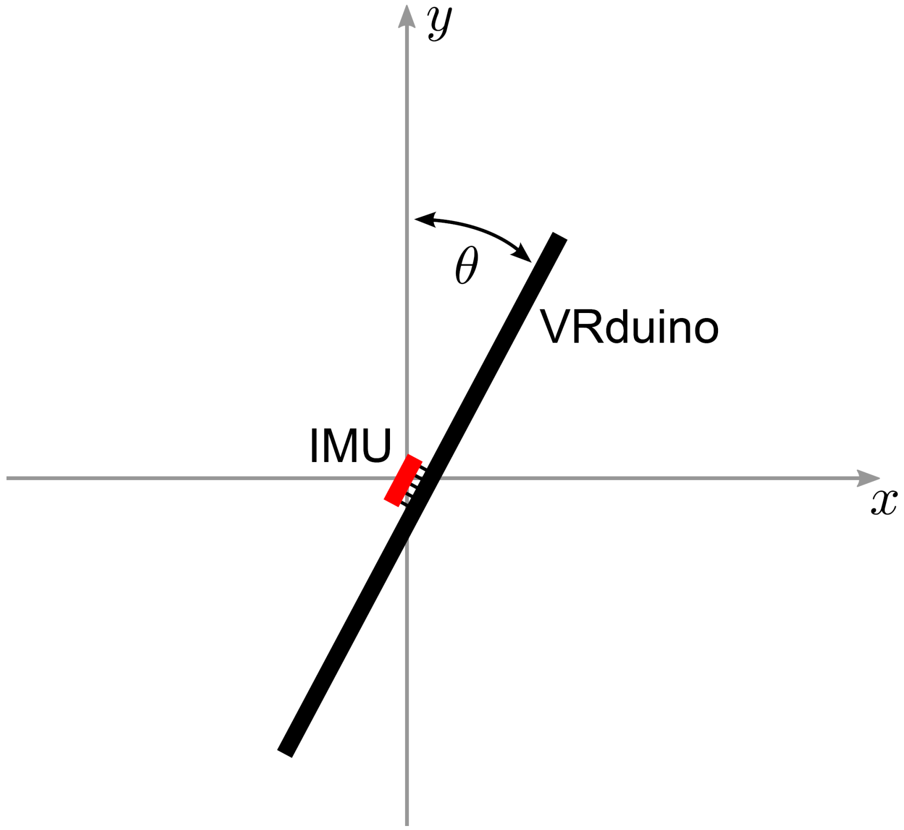
Orientation Tracking Example

- Gyro integration

$$\theta_{gyro}^{(t)} = \theta_{gyro}^{(t-1)} + \tilde{\omega}\Delta t$$

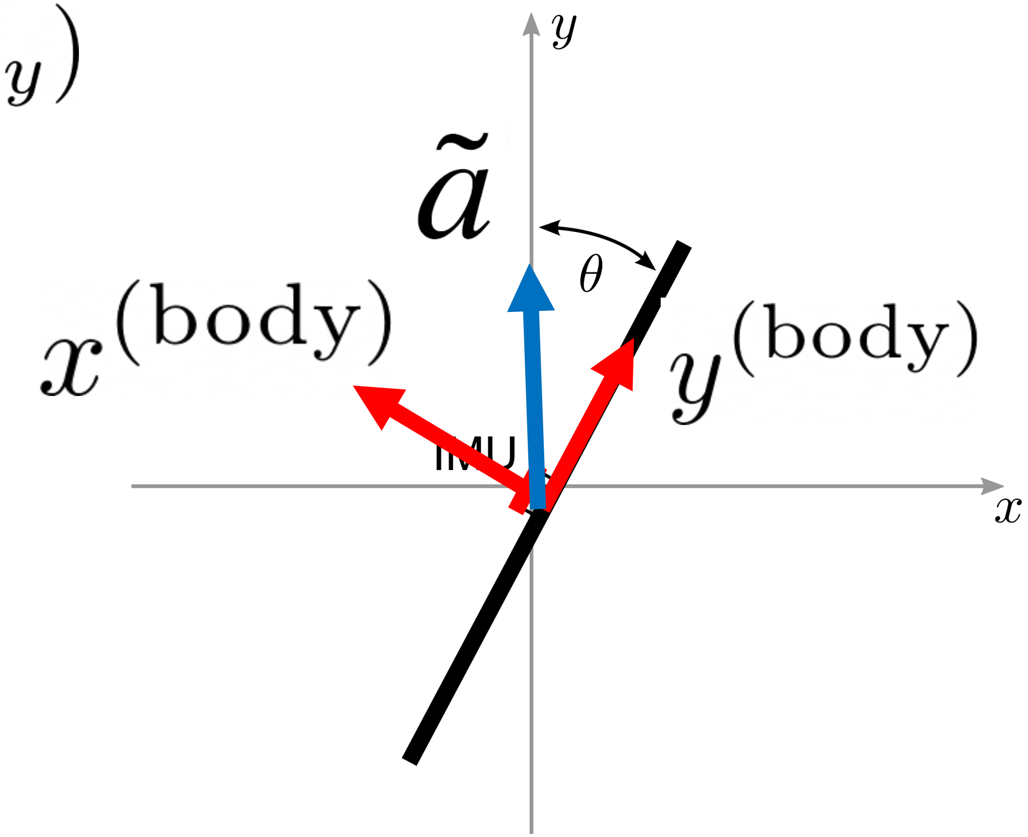
$$\theta_{gyro}^{(0)} = 0$$

Problem: Drift



Orientation Tracking Example

- Angle from accelerometers
  - Measurements in body $\tilde{a} = (\tilde{a}_x, \tilde{a}_y)$
  - Corresponds to gravity in world



$$\theta_{acc} = \tan^{-1}\left(\frac{\tilde{a}_x}{\tilde{a}_y}\right)$$

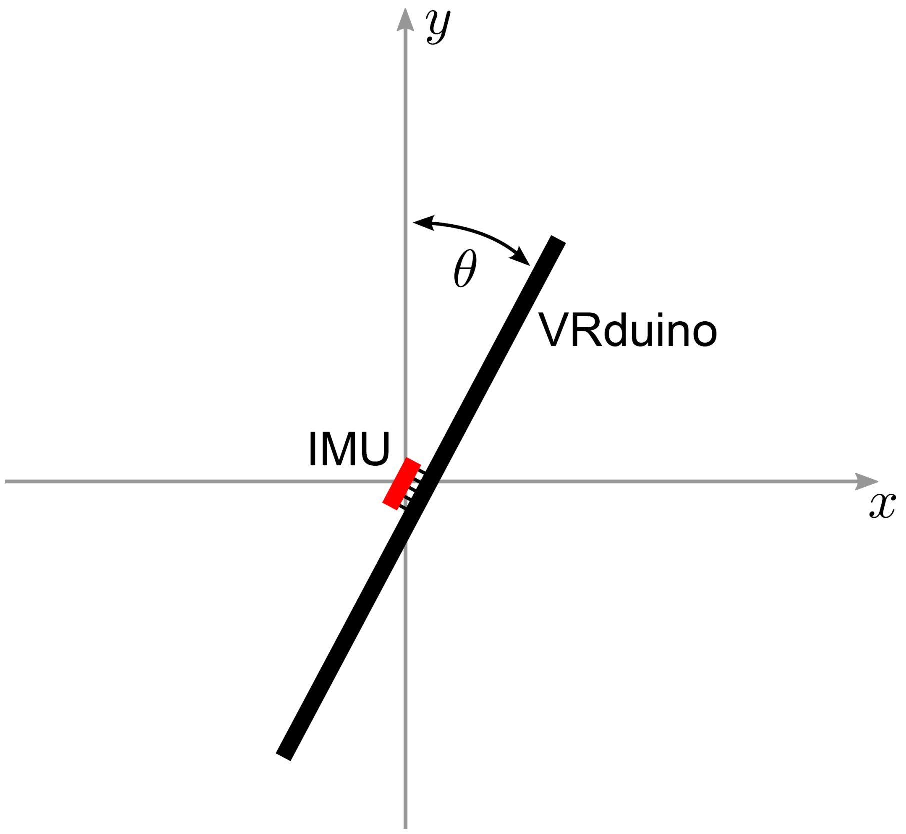
Problem: noises

Orientation Tracking Example

- Sensor fusion
  - Combine gyro and accelerometer measurements

$$\theta^{(t)} = \alpha\left(\theta^{(t-1)} + \tilde{\omega}\Delta t\right) + (1-\alpha)\,\text{atan2}\left(\tilde{a}_x, \tilde{a}_y\right)$$

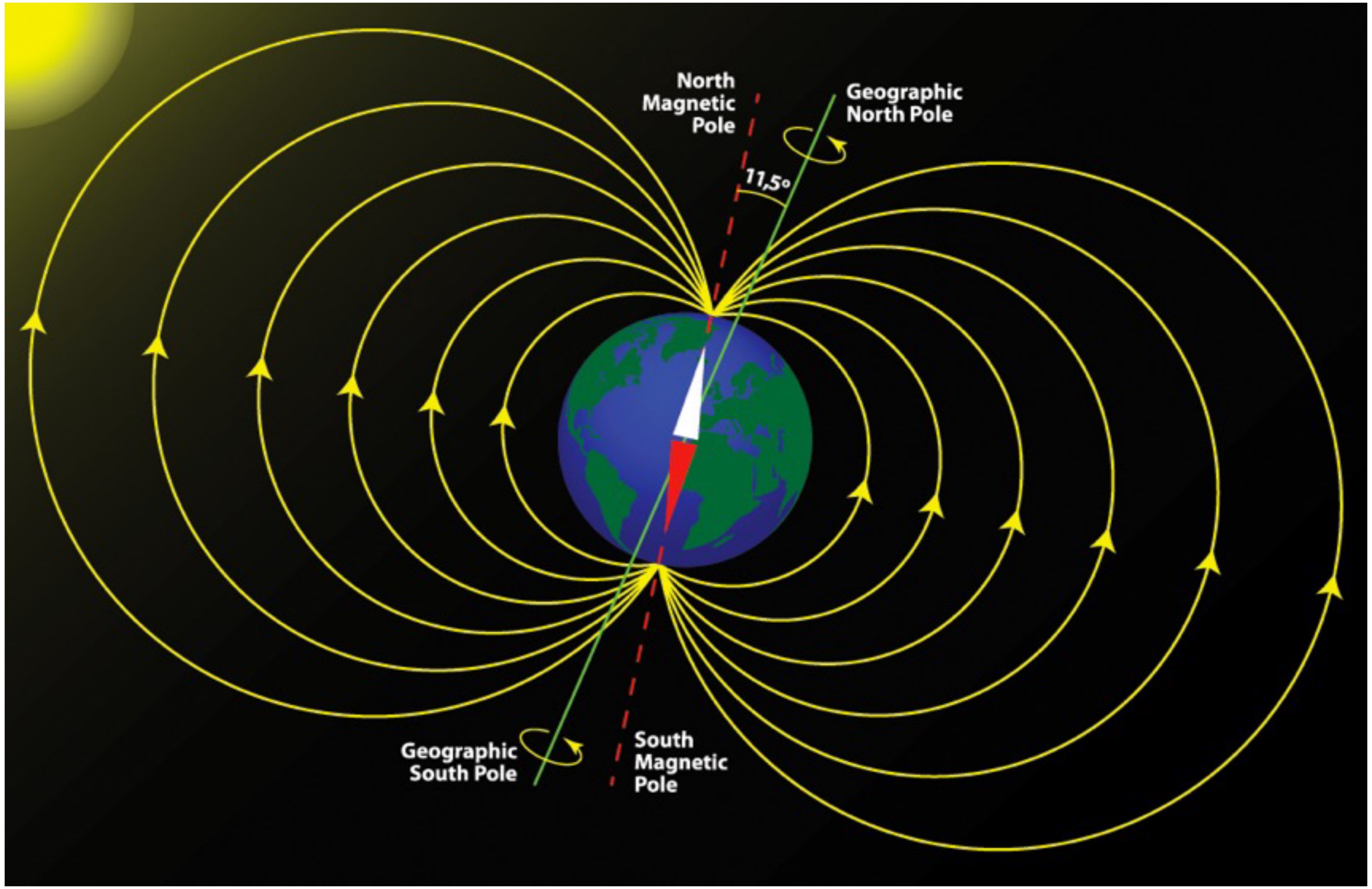
complementary filter



Magnetometers

wikipedia

Compass



lifescience.com

Magnetometers

- Measure earth’s magnetic field in Gauss or uT (micro Tesla)
- 3 orthogonal axes
  - Vector pointing along the magnetic field
- Actual direction depends on latitude and longitude
- Distortions due to metal or electronics objects in room or in HMD

Magnetometers

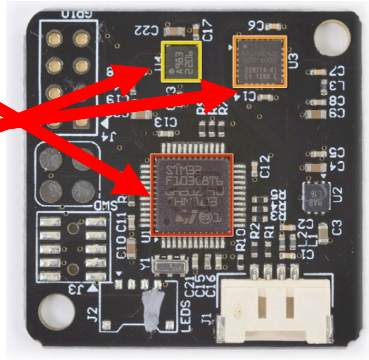
- Pros
  - Complementary to accelerometers

- Cons
  - Affected by metal, distortions of magnetic field
  - Need to know location even when calibrated (e.g., GPS)

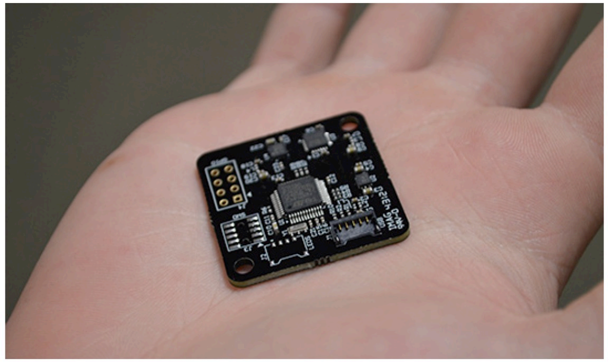
- Together with gyros and accelerometers, 9 DOF sensor fusion

Oculus Rift

- STMicroelectronics 32F103C8 ARM Cortex-M3 microcontroller
- Invensense MPU-6000 (gyroscope + accelerometer)
- Honeywell HMC5983 magnetometer

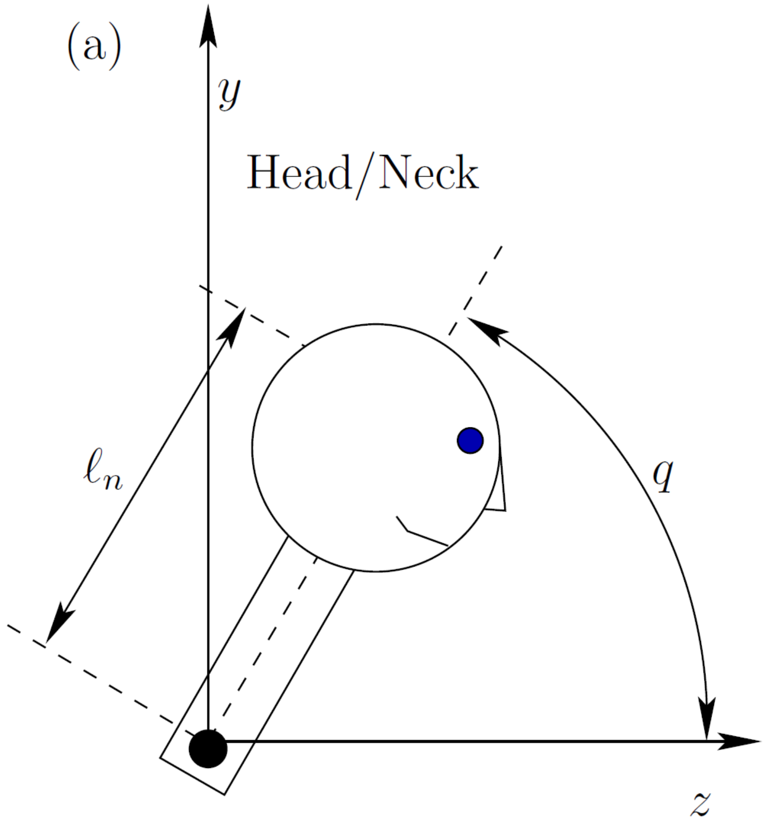


- 3-axis gyro, 3-axis accelerometer, 3-axis magnetometer all on one chip



S. LaValle, A. Yershova, M. Katsev, M. Antonov. Head Tracking for the Oculus Rift. ICRA’14

Head Position Tracking in Oculus Rift



$$p = f(q)$$

Head position

Quaternion of head orientation

$$p = f(q) = q * (0, \ell_n, 0) * q^{-1}$$

Use quaternion to rotate a vector

S. LaValle, A. Yershova, M. Katsev, M. Antonov. Head Tracking for the Oculus Rift. ICRA’14

Further Reading

- Sections 9.1, 9.2 in Virtual Reality, Steven LaValle

- Stanford EE 267 course note on 3DOF orientation tracking and IMUs
  - https://stanford.edu/class/ee267/notes/ee267_notes_imu.pdf

- Head Tracking for the Oculus Rift
  - http://msl.cs.illinois.edu/~lavalle/papers/LavYerKatAnt14.pdf