



THE UNIVERSITY OF TEXAS AT DALLAS

Image Processing: Filtering II

CS 4391 Introduction to Computer Vision

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Many slides in this lecture were inspired or adapted from Ioannis (Yannis) Gkioulekas.

Filtered Image (Gaussian)



Noisy Image




Question: How to handle blurry artifacts and preserve image edges in the filtered image?

Recap: Image Filtering

Modify the pixels in an image based on some function of a local neighborhood of each pixel

10	5	3
4	5	1
1	1	7

Local image data

Some function


	7	

Modified image data

Let f be the image, w be the $(2n + 1) \times (2n + 1)$ kernel weights and h be the filtered output image

$$h[u, v] = \sum_{k=-n}^n \sum_{l=-n}^n w[k, l] f[u + k, v + l]$$

Recap: Image Filtering Process

$1/9$	$1/9$	$1/9$
$1/9$	$1/9$	$1/9$
$1/9$	$1/9$	$1/9$

kernel



Noisy Image

Apply the filter to every pixel

Recap: Image Filtering Process

$1/9$	$1/9$	$1/9$
$1/9$	$1/9$	$1/9$
$1/9$	$1/9$	$1/9$

kernel

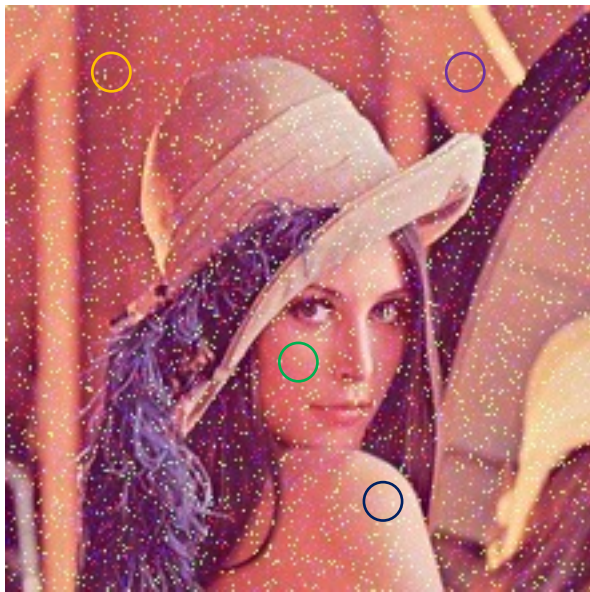


Filtered Image

Apply the filter to every pixel

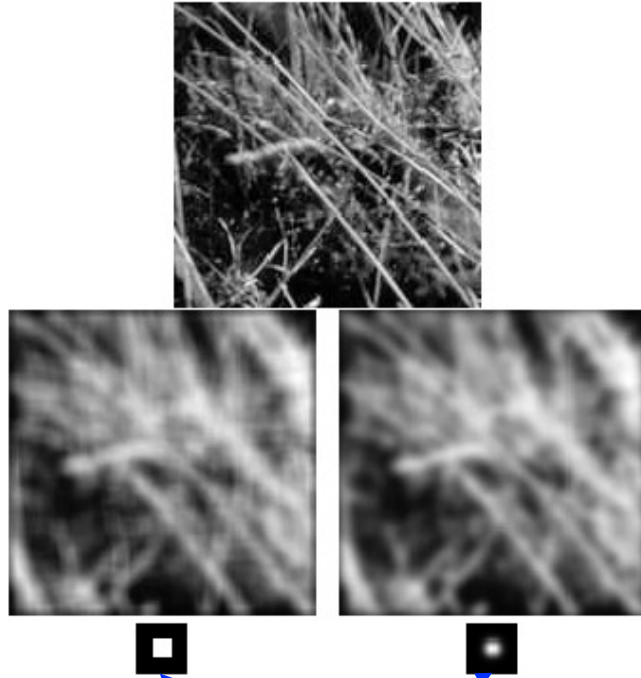
Recap: Image Prior: Local Smoothness

- Local natural image regions are typically smooth or uniform
- The overall structures or texture of a natural image often has a more subtle and gradual variation than image noise



- Image pixels in a small window (e.g., 5x5) usually are similar
- Noise values are dramatically changing at arbitrary directions
- Due to noises, a noisy image have higher local variations than the clean image

Recap: Local Smoothness with Mean vs Gaussian filtering



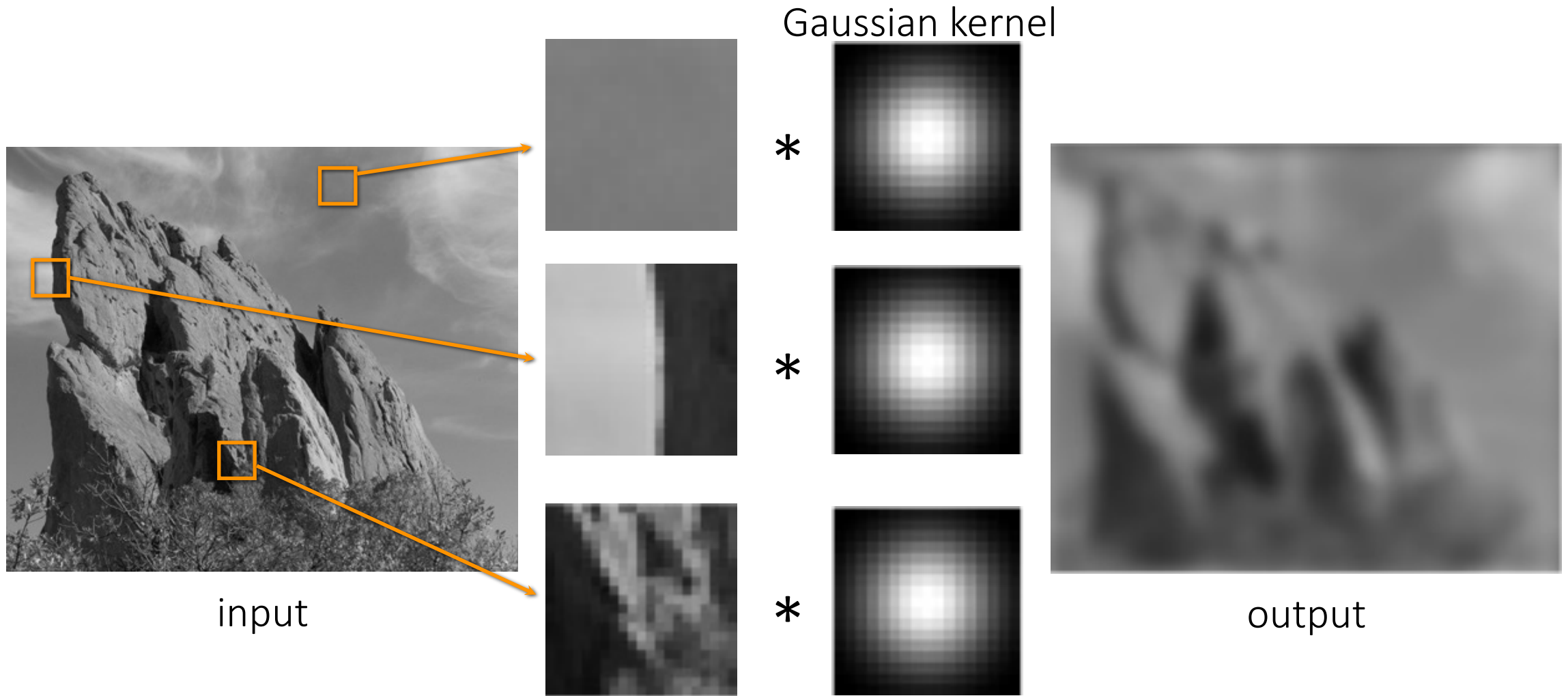
Both mean and Gaussian utilize local smoothness prior

- Mean filter assumes all pixels in a local window are equally important
- Gaussian filter assumes pixels that are closer to the target pixel are more important

We need to design a better kernel w for improving filtering results.

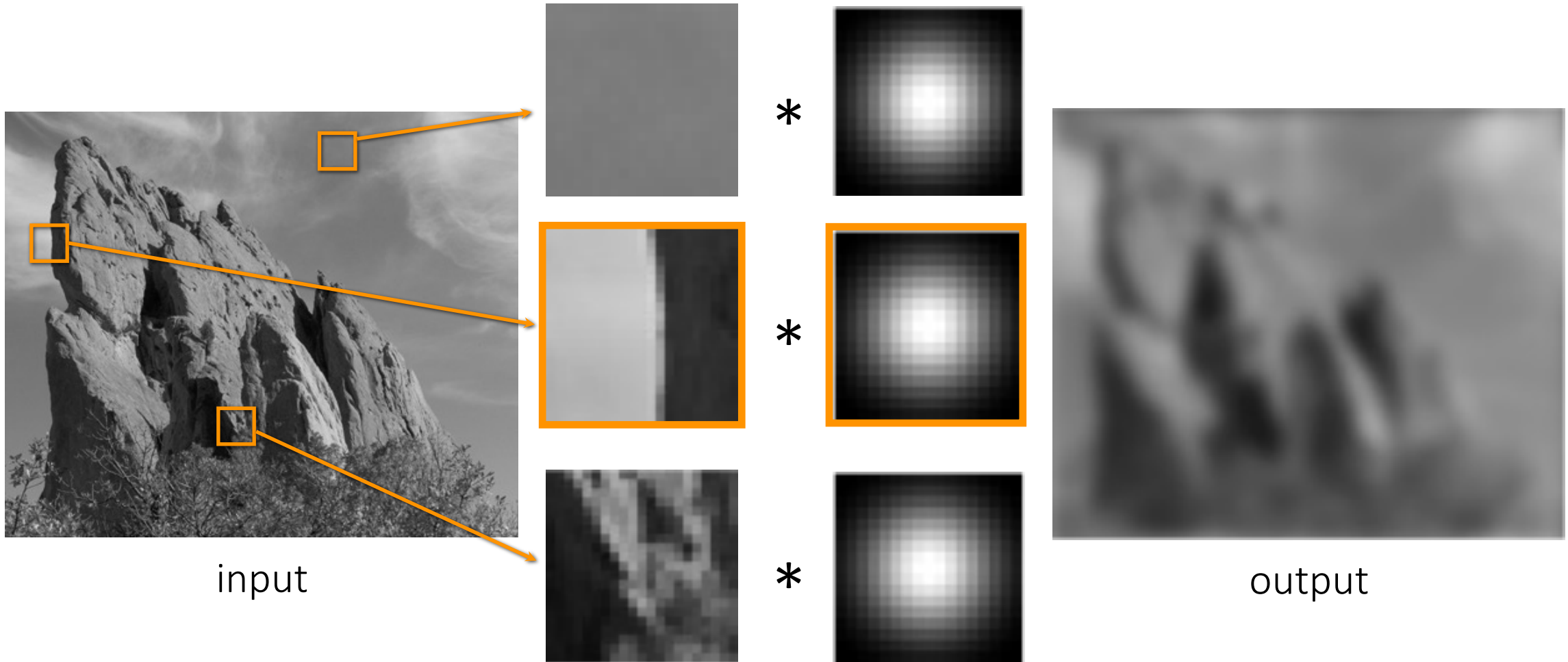
$$h[u, v] = \sum_{k=-n}^n \sum_{l=-n}^n w[k, l] f[u + k, v + l]$$

The problem with Gaussian filtering



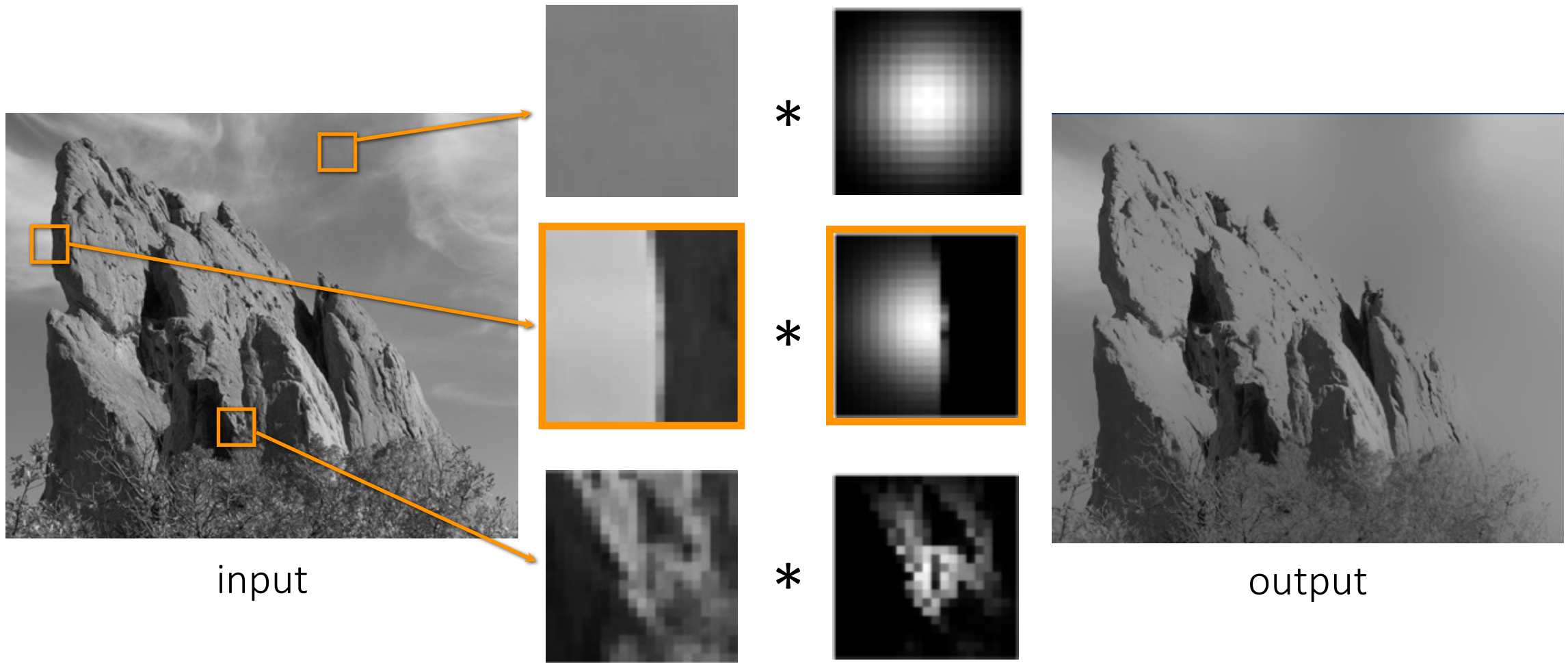
The problem with Gaussian filtering

Gaussian kernel



The bilateral filtering solution: Edge-preserving local smoothness

bilateral filter kernel

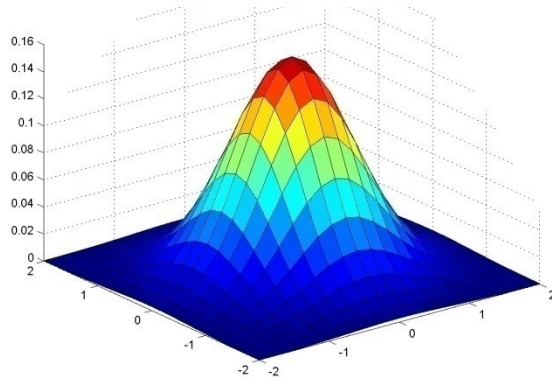


Bilateral filtering

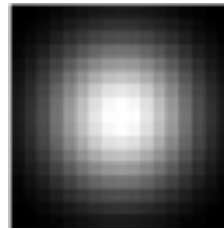
$$h[m, n] = \frac{1}{W_{mn}} \sum_{k,l} g[k, l] r_{mn}[k, l] f[m + k, n + l]$$

Bilateral filtering

$$h[m, n] = \frac{1}{W_{mn}} \sum_{k,l} g[k, l] r_{mn}[k, l] f[m + k, n + l]$$



Spatial weighting



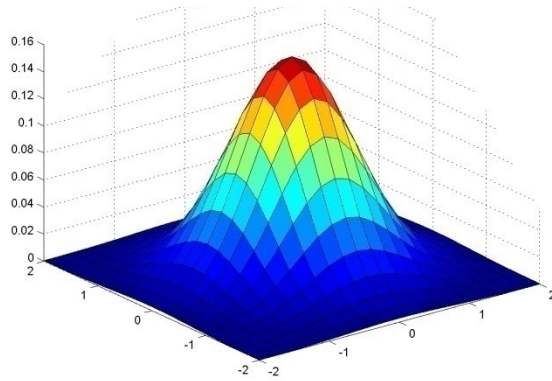
$$g[k, l] = \frac{1}{2\pi\sigma_s^2} \exp\left(-\frac{(k^2 + l^2)}{2\sigma_s^2}\right)$$

σ_s

Assign a pixel a large weight if: 1) it's nearby

Bilateral filtering

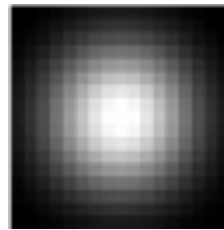
$$h[m, n] = \frac{1}{W_{mn}} \sum_{k,l} g[k, l] r_{mn}[k, l] f[m + k, n + l]$$



$$g[k, l] = \frac{1}{2\pi\sigma_s^2} \exp\left(-\frac{(k^2 + l^2)}{2\sigma_s^2}\right)$$

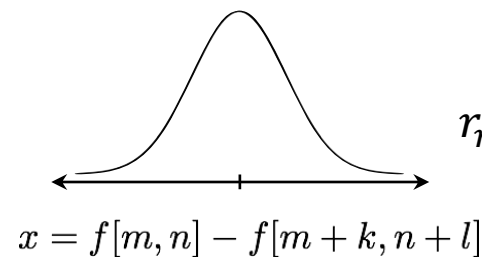
Assign a pixel a large weight if:

Spatial weighting



σ_s

Intensity range weighting



σ_r

$$r_{mn} = \frac{1}{\sqrt{2\pi}\sigma_r} e^{-\frac{x^2}{2\sigma_r^2}}$$

1) it's nearby and 2) it looks like me

Bilateral filtering

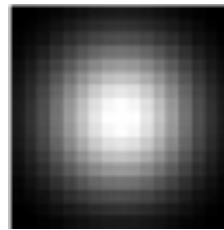
$$h[m, n] = \frac{1}{W_{mn}} \sum_{k,l} g[k, l] r_{mn}[k, l] f[m + k, n + l]$$

Normalization factor

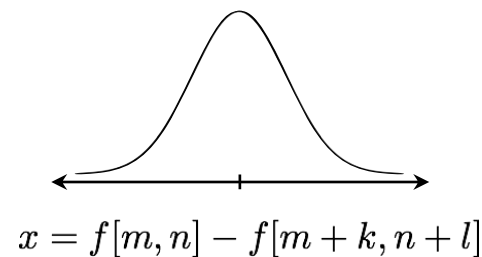
Spatial weighting

Intensity range weighting

$$W_{mn} = \sum_{k,l} g[k, l] r_{mn}[k, l]$$



σ_s



σ_r

Assign a pixel a large weight if: 1) it's nearby and 2) it looks like me

Implementation: Bilateral filtering

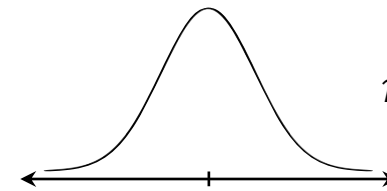
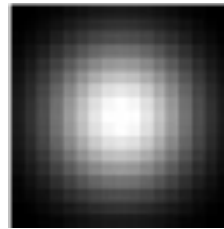
$$h[m, n] = \frac{1}{W_{mn}} \sum_{k,l} g[k, l] r_{mn}[k, l] f[m + k, n + l]$$

Normalization factor

Spatial weighting

Intensity range weighting

$$W_{mn} = \sum_{k,l} g[k, l] r_{mn}[k, l]$$



$$r_{mn} = \frac{1}{\sqrt{2\pi}\sigma_r} e^{-\frac{x^2}{2\sigma_r^2}}$$

$$g[k, l] = \frac{1}{2\pi\sigma_s^2} \exp\left(-\frac{(k^2 + l^2)}{2\sigma_s^2}\right)$$

σ_s

$$x = f[m, n] - f[m + k, n + l]$$

σ_r

Bilateral filtering vs Gaussian filtering

Which is which?

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$

$$h[m, n] = \frac{1}{W_{mn}} \sum_{k, l} g[k, l] r_{mn}[k, l] f[m + k, n + l]$$

Bilateral filtering vs Gaussian filtering

Gaussian filtering

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$

Bilateral filtering

$$h[m, n] = \frac{1}{W_{mn}} \sum_{k, l} g[k, l] r_{mn}[k, l] f[m + k, n + l]$$

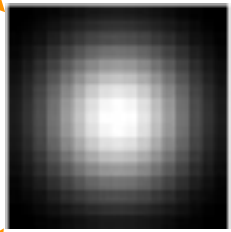
Bilateral filtering vs Gaussian filtering

Gaussian filtering

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$

Bilateral filtering

$$h[m, n] = \frac{1}{W_{mn}} \sum_{k, l} g[k, l] r_{mn}[k, l] f[m + k, n + l]$$

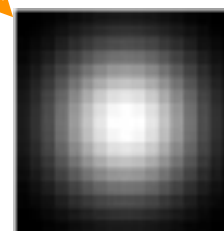
σ_s  Spatial weighting:
favor *nearby* pixels

Bilateral filtering vs Gaussian filtering

Gaussian filtering

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$

σ_s

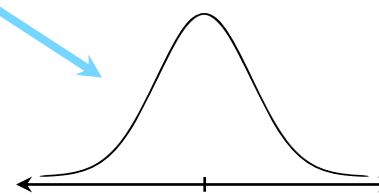


Spatial weighting:
favor *nearby* pixels

Bilateral filtering

$$h[m, n] = \frac{1}{W_{mn}} \sum_{k, l} g[k, l] r_{mn}[k, l] f[m + k, n + l]$$

σ_r



Intensity range weighting:
favor *similar* pixels

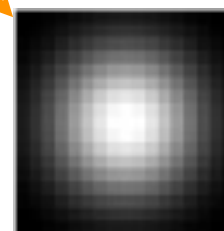
$$x = f[m, n] - f[m + k, n + l]$$

Bilateral filtering vs Gaussian filtering

Gaussian filtering

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$

σ_s



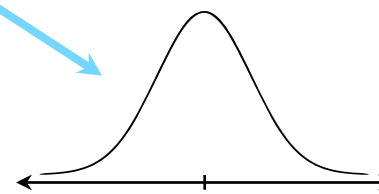
Spatial weighting:
favor *nearby* pixels

Bilateral filtering

$$h[m, n] = \frac{1}{W_{mn}} \sum_{k, l} g[k, l] r_{mn}[k, l] f[m + k, n + l]$$

Normalization factor

σ_r



Intensity range weighting:
favor *similar* pixels

$$x = f[m, n] - f[m + k, n + l]$$

Bilateral filtering vs Gaussian filtering

Gaussian filtering

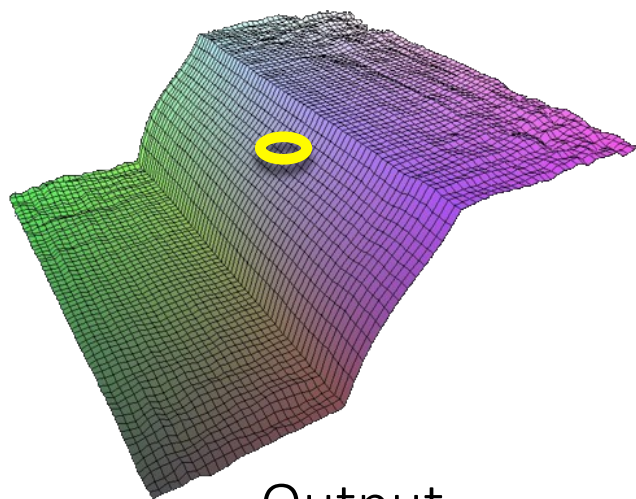
Smooths everything nearby (even edges)
Only depends on *spatial* distance

Bilateral filtering

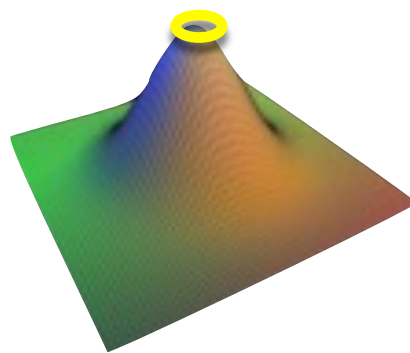
Smooths 'close' pixels in space and intensity
Depends on *spatial* and *intensity* distance

Gaussian filtering visualization

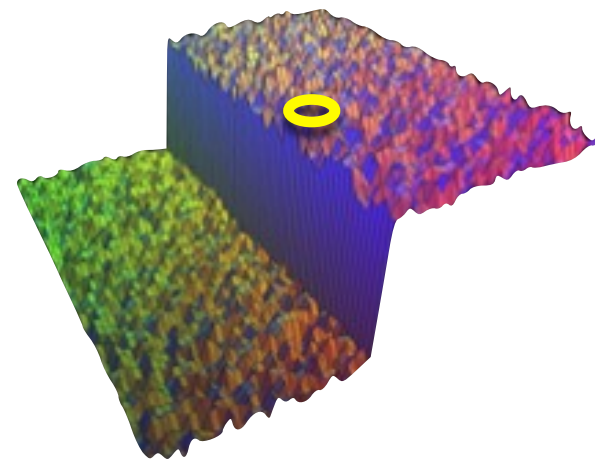
$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$



Output



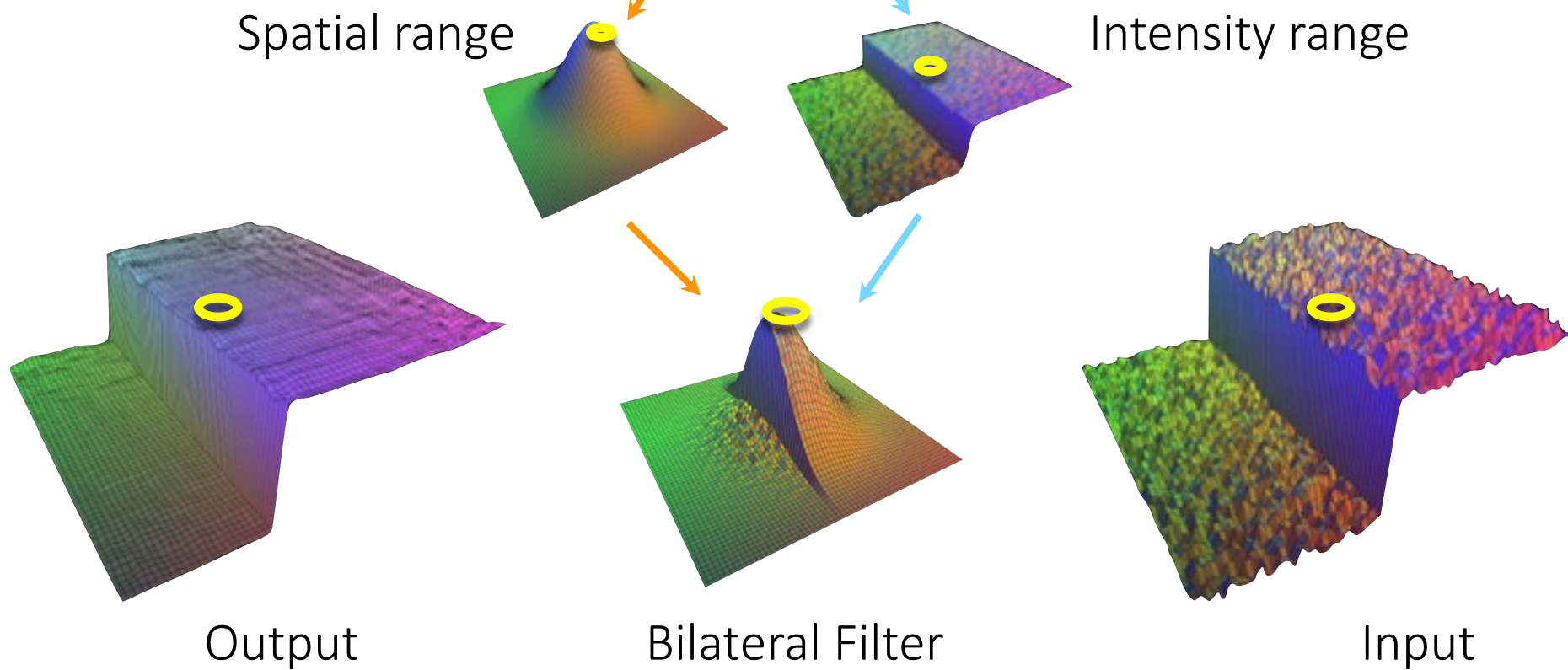
Gaussian Filter



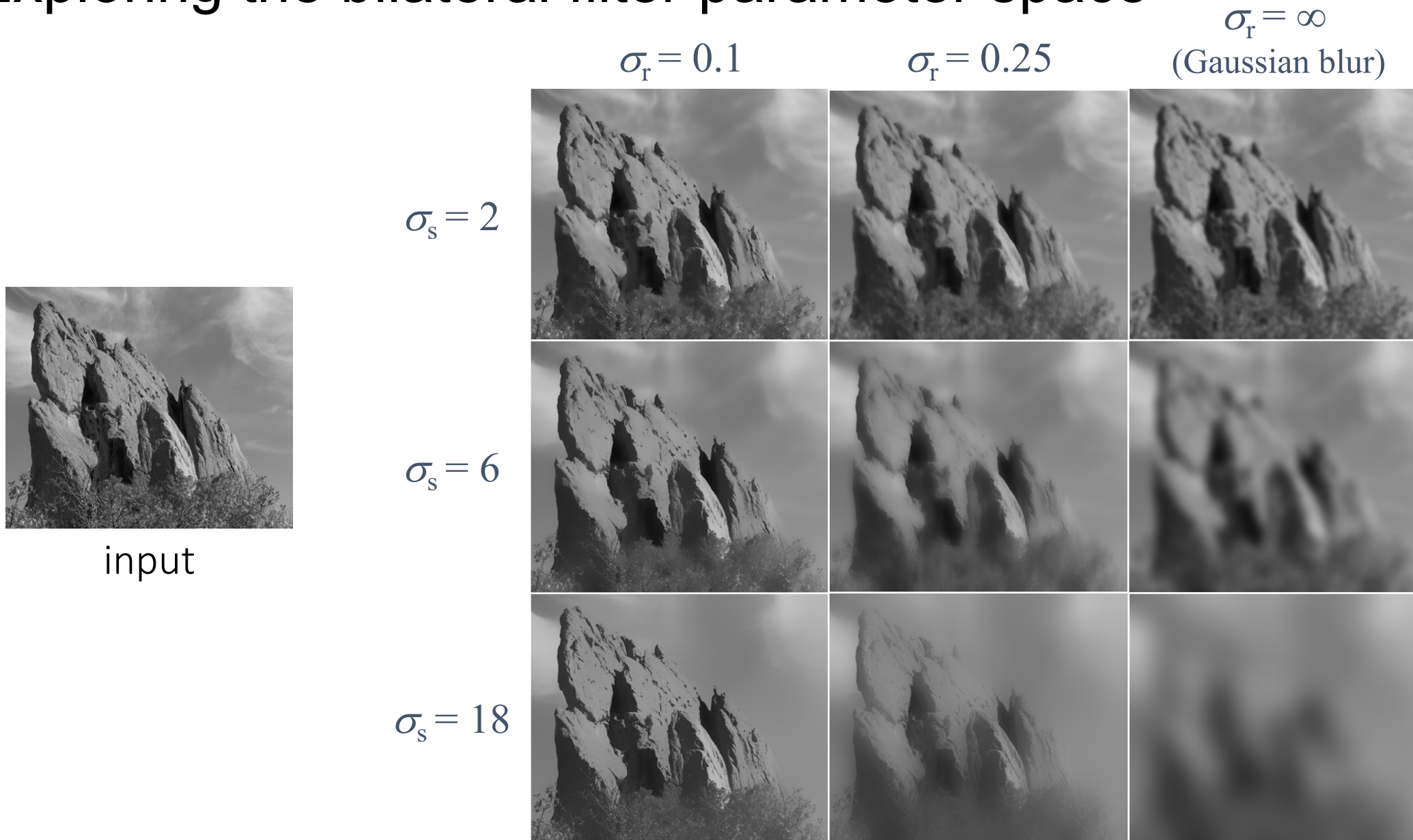
Input

Bilateral filtering visualization

$$h[m, n] = \frac{1}{W_{mn}} \sum_{k, l} g[k, l] r_{mn}[k, l] f[m + k, n + l]$$

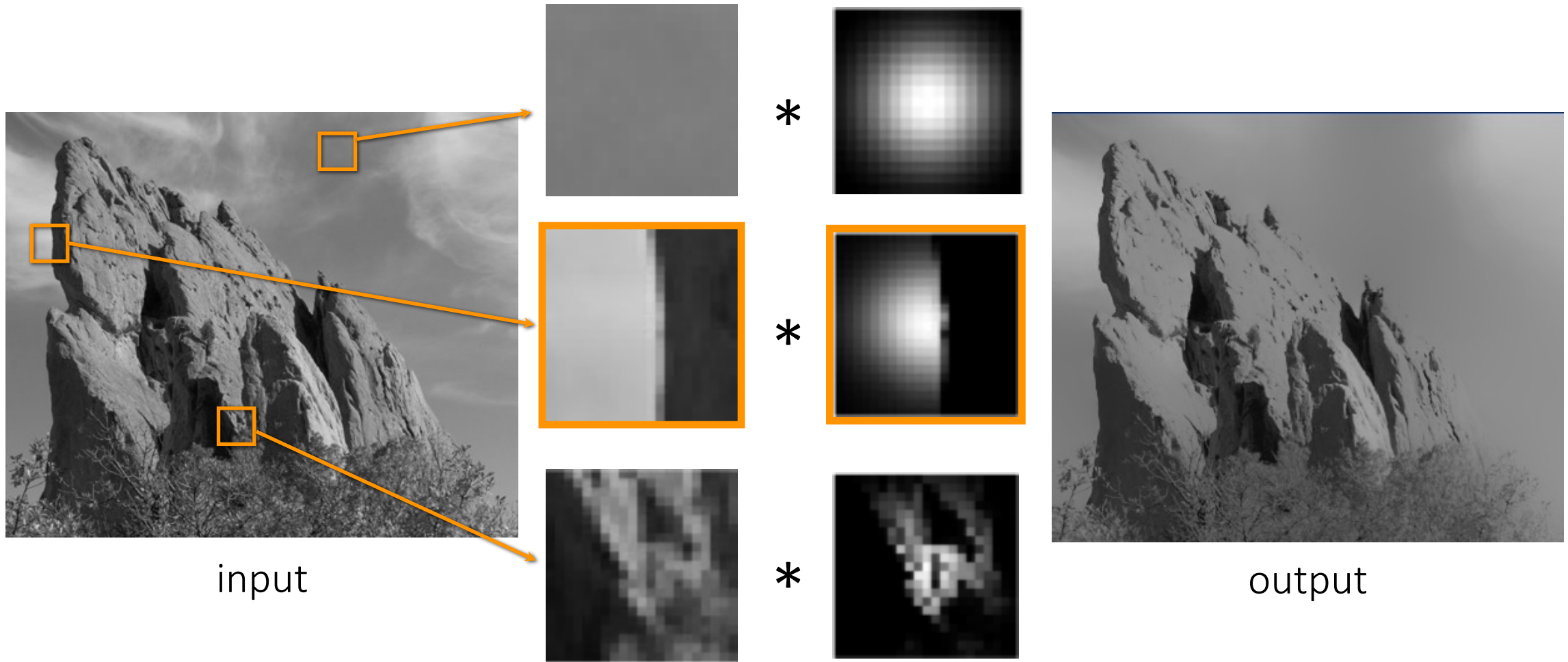


Exploring the bilateral filter parameter space

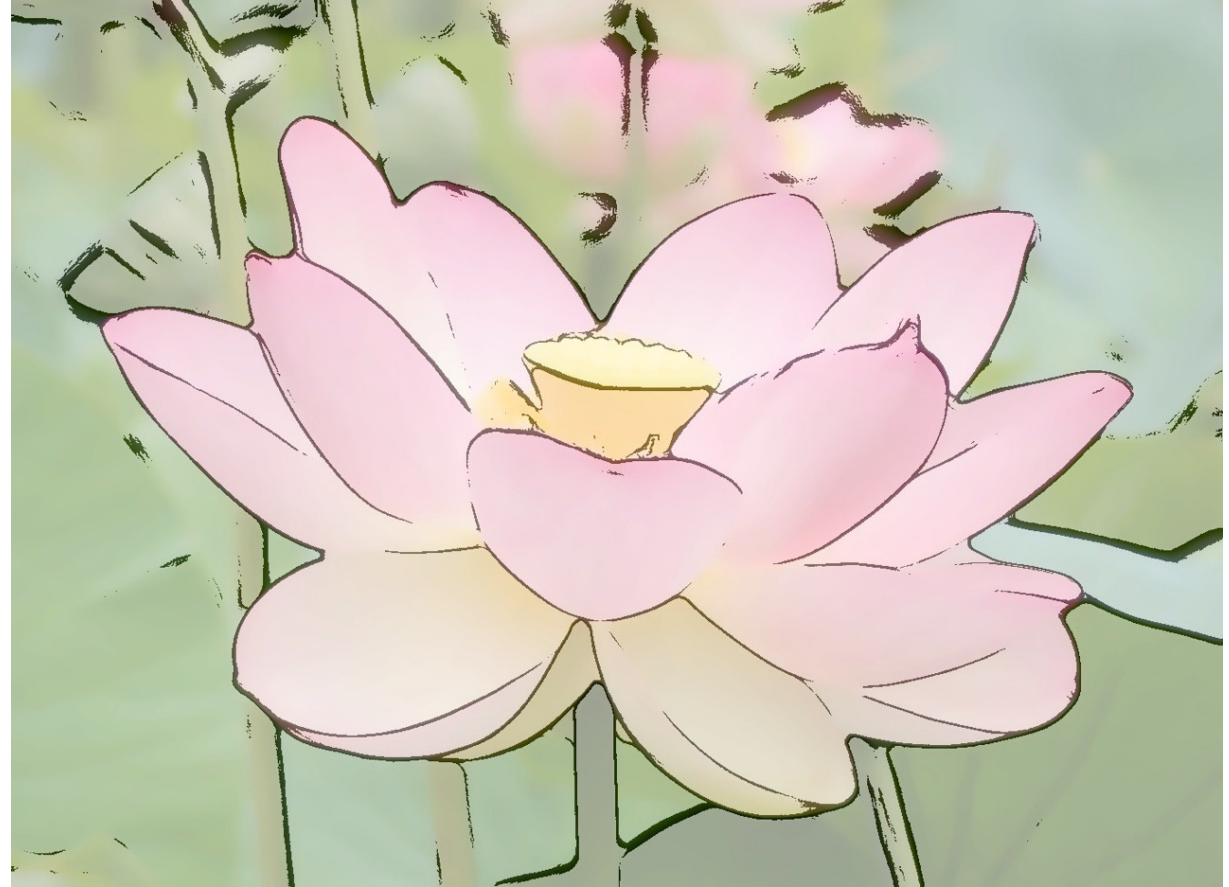


The bilateral filtering solution

bilateral filter kernel

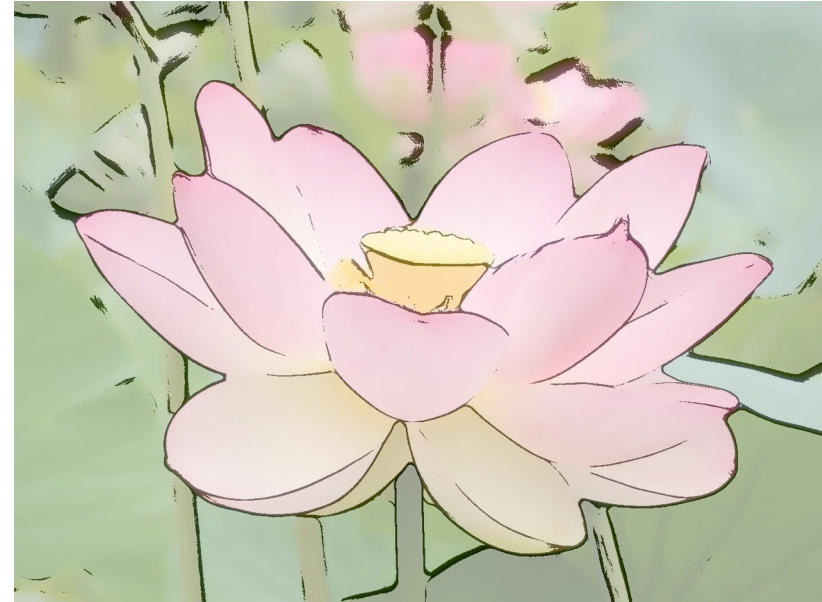


Application: Cartoonization



How would you create this effect?

Application: Cartoonization



edges from bilaterally filtered image



+

bilaterally filtered image



=

cartoon rendition

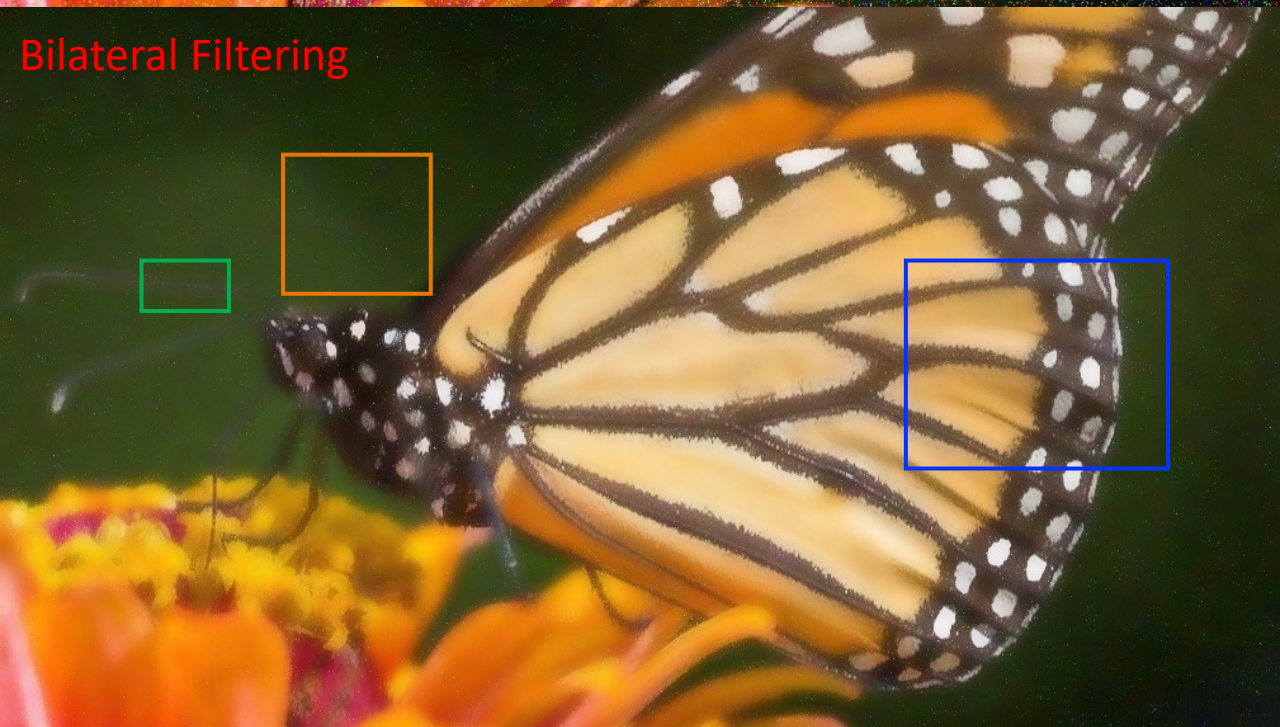




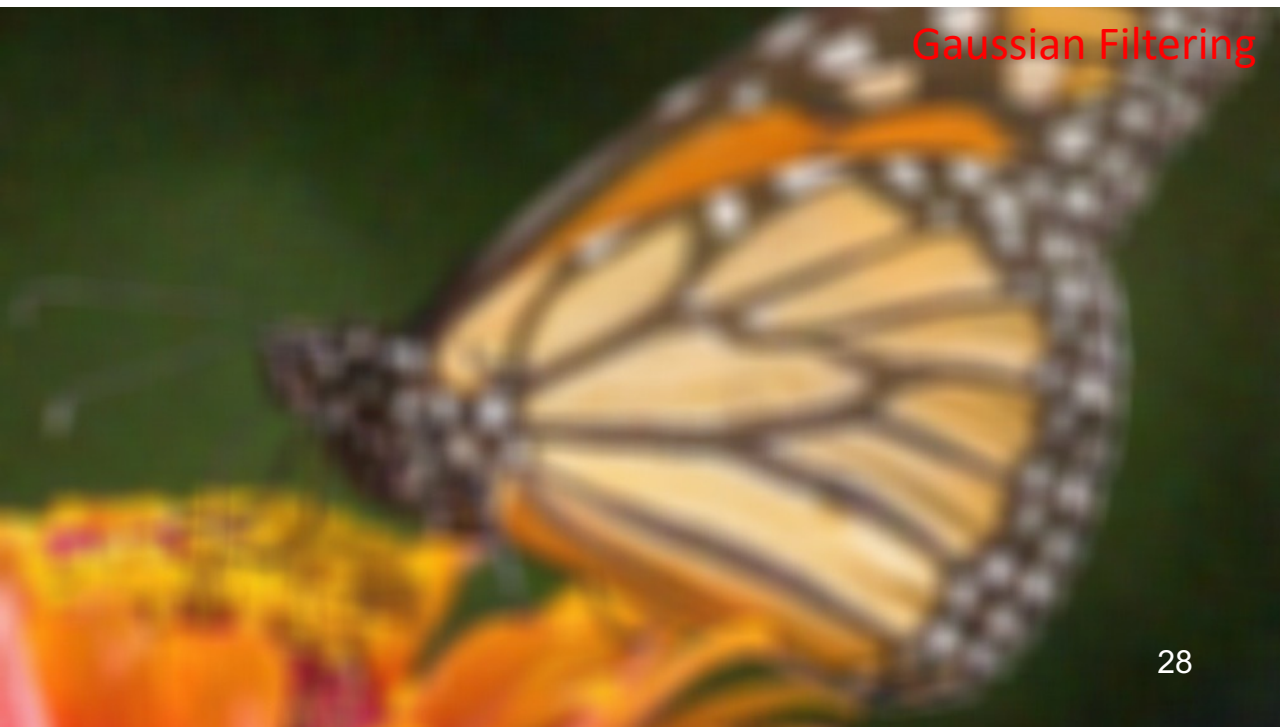
Noisy Image

Application: Image Denoising with Bilateral Filtering

- Sharper edges
- Some thin edges may be reduced
- Flat regions are not fully smoothed

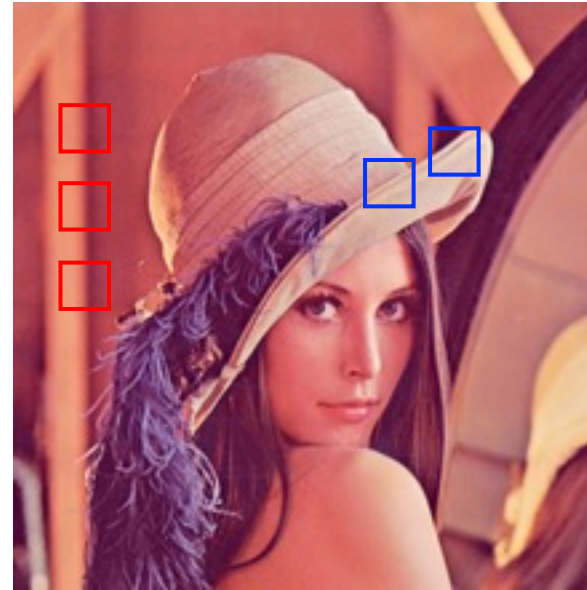
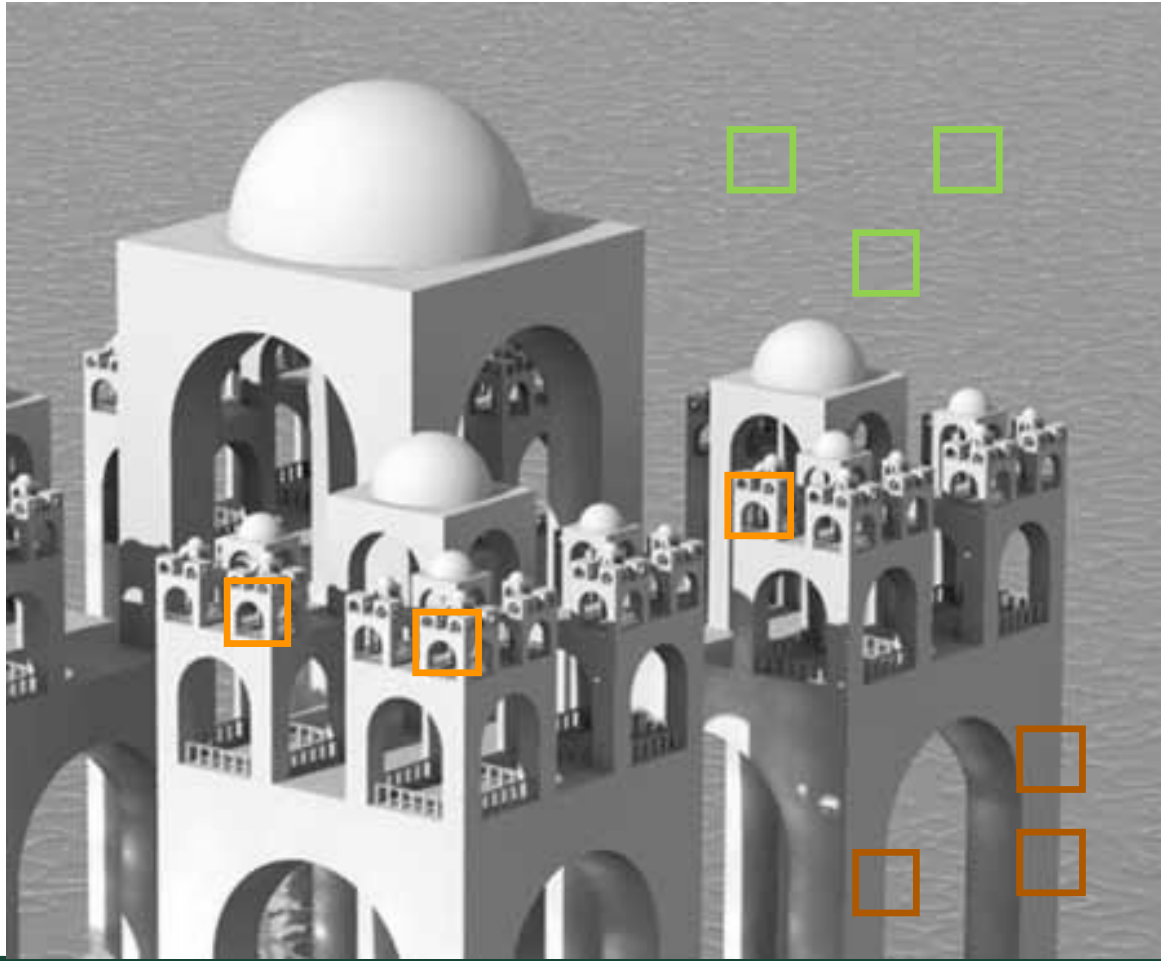


Bilateral Filtering



Gaussian Filtering

Image Prior: Non-local smoothness/redundancy



Small patches in natural images tend to redundantly appear multiple times

Non-local means Filter

No need to stop at neighborhood. Instead search *everywhere* in the image.

Given a pixel $f(p)$ at position $p = (p_x, p_y)$, the filter uses pixels in the whole image to update $f(p)$

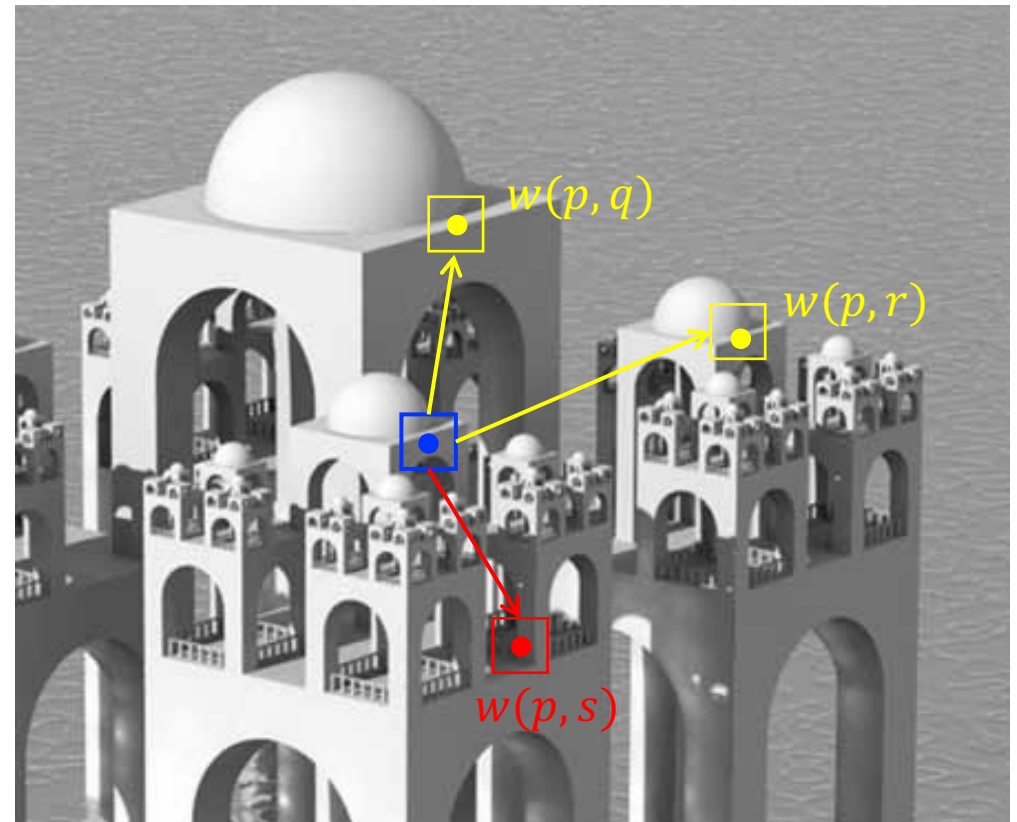
$$h(p) = \frac{1}{W} \sum_q w(p, q) f(q)$$

Weight: $w(p, q) = \exp\left(-\frac{SSD(p, q)}{2\sigma^2}\right)$

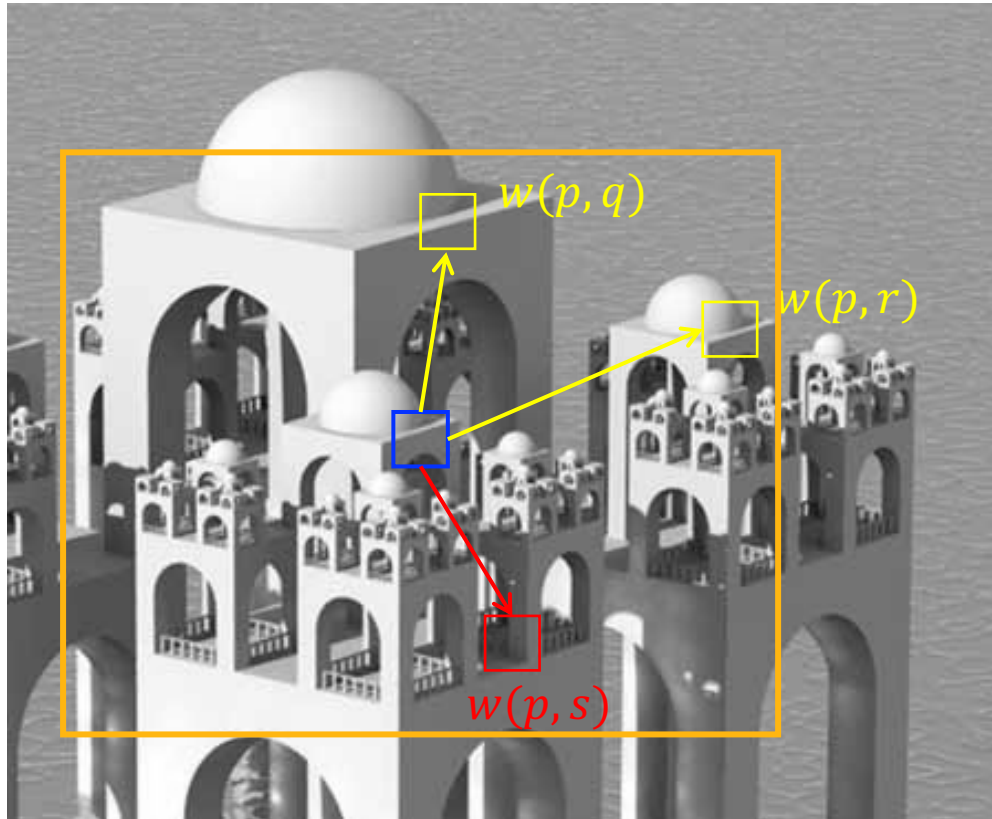
Sum of the squared difference between two patches

$$SSD(p, q) = \sum_{k=-n}^n \sum_{l=-n}^n (f(p_x + k, p_y + l) - f(q_x + k, q_y + l))^2$$

$W = \sum_q w(p, q)$ is the normalization term



Fast Implementation of Non-local Means



Scan over the whole image to compute weights for each pixel is time-consuming
Implementation:

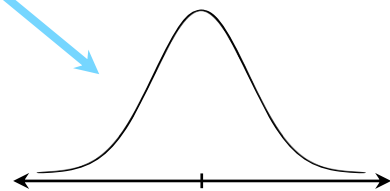
- set a search window (e.g., 21x21) with the target pixel position as the center
- only use pixels inside the window to compute weights based on patch similarity

Patch size (e.g., 5x5, 7x7) is much smaller than the window size

Non-local means vs bilateral filtering

Non-local means filtering

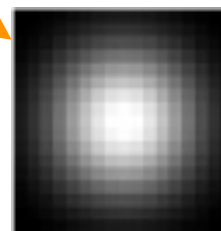
$$h[m, n] = \frac{1}{W_{mn}} \sum_{k,l} r_{mn}[k, l] f[m + k, n + l]$$



Intensity range weighting:
favor *similar* pixels (patches
in case of non-local means)

Bilateral filtering

$$h[m, n] = \frac{1}{W_{mn}} \sum_{k,l} g[k, l] r_{mn}[k, l] f[m + k, n + l]$$



Spatial weighting:
favor *nearby* pixels

Nonlocal Means Filtering



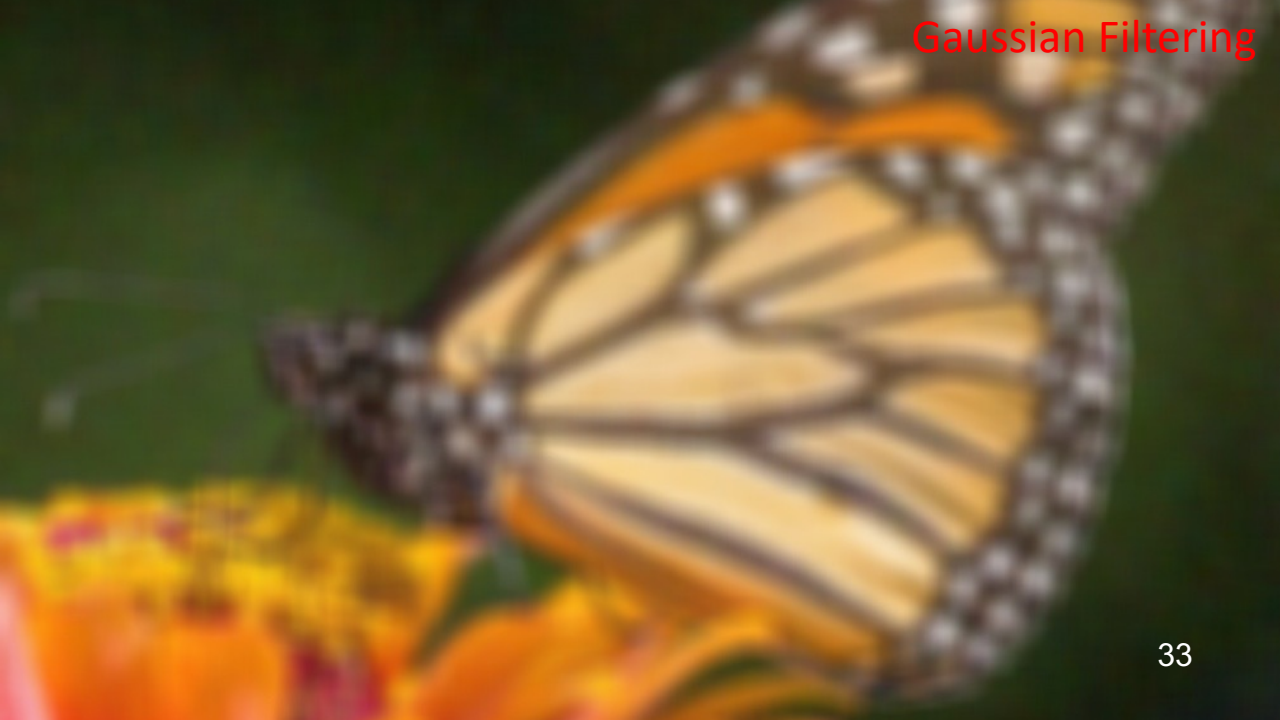
Noisy Image



Bilateral Filtering



Gaussian Filtering



Summary

Gaussian filtering

Smooths everything nearby (even edges)
Only depends on *spatial* distance

Bilateral filtering

Smooths 'close' pixels in space and intensity
Depends on *spatial* and *intensity* distance

Non-local means

Smooths similar patches no matter how far away
Only depends on *intensity* distance

Further Reading

Chapters 3.3.1 and 3.3.2, Computer Vision: Algorithms and Applications, Richard Szeliski

https://en.wikipedia.org/wiki/Non-local_means