THE UNIVERSITY OF TEXAS AT DALLAS

Recurrent Neural Networks

CS 4391 Introduction to Computer Vision
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Department of Computer Science

Slides borrowed from Professor Yu Xiang

Single Images

Convolutional neural networks

Image

CNN

High-level information
- Depth
- Object classes
- Object poses
- Etc.

Sequential Data

Data depends on time

- Video

t-1

t

t+1

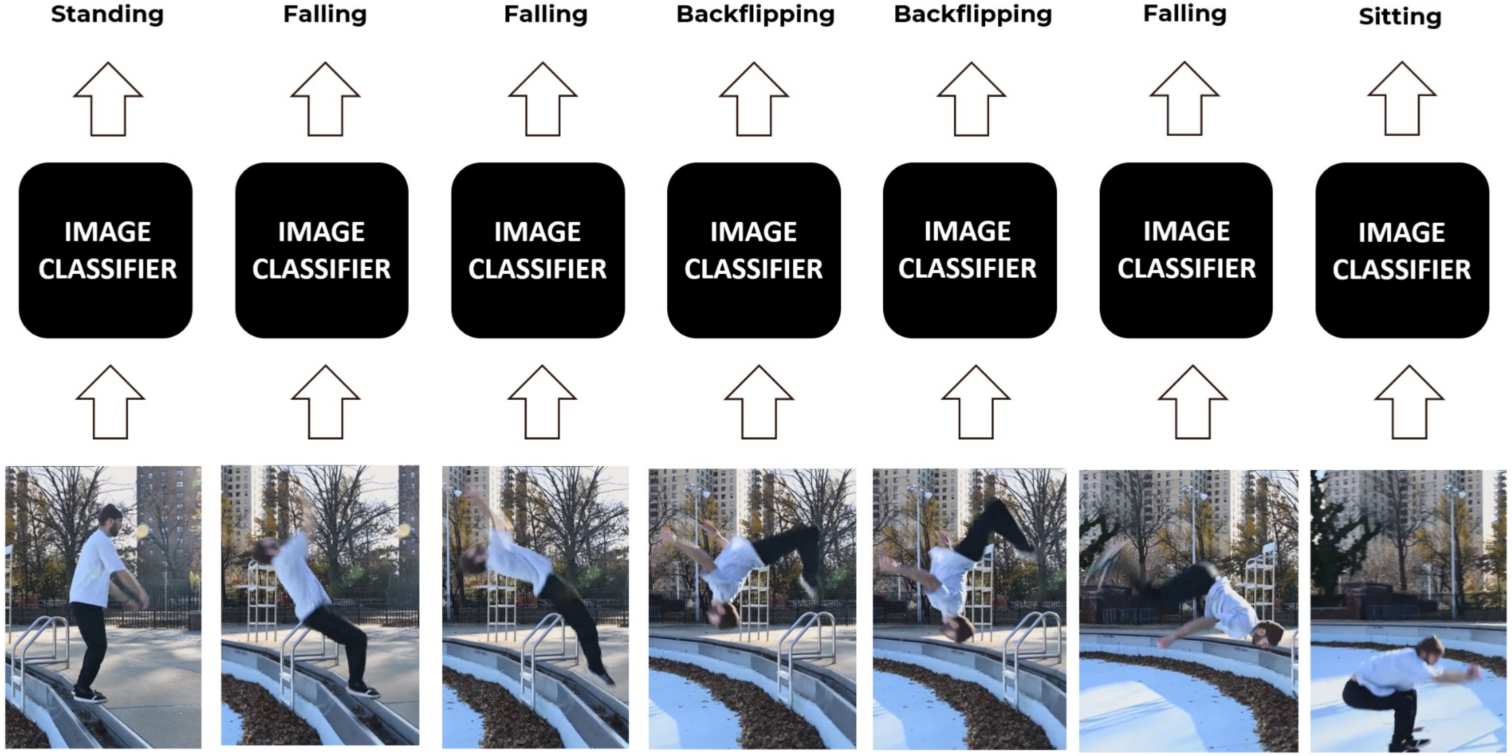
- Sentence

UT Dallas is a rising public research university in the heart of DFW.

t

Sequential Data Labeling

Video frame labeling

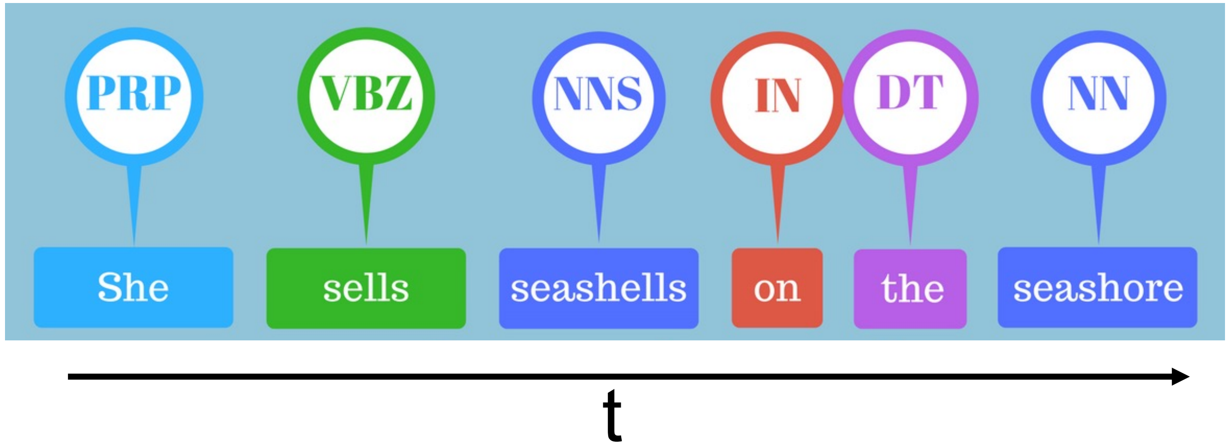


Frames of a Video

https://bleedai.com/human-activity-recognition-using-tensorflow-cnn-lstm/

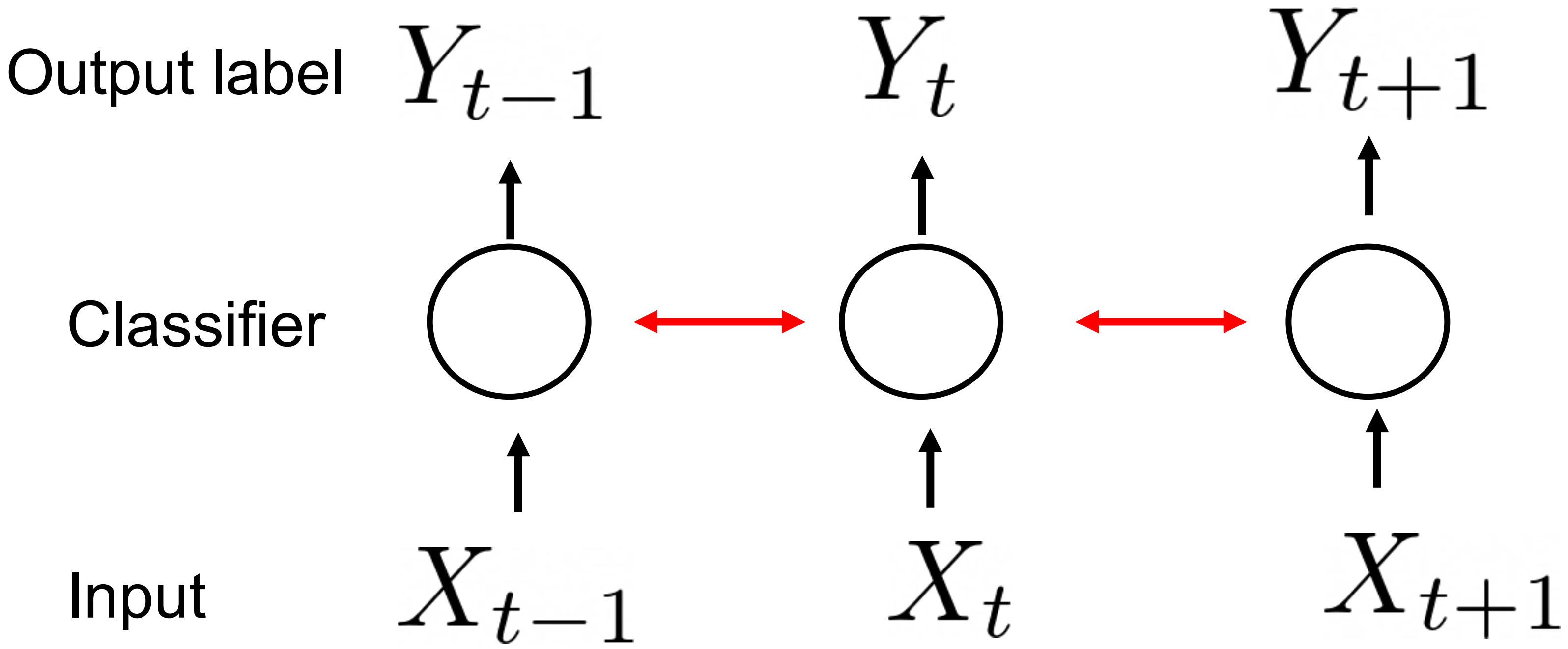
Sequential Data Labeling

Part-of-speech tagging (grammatical tagging)



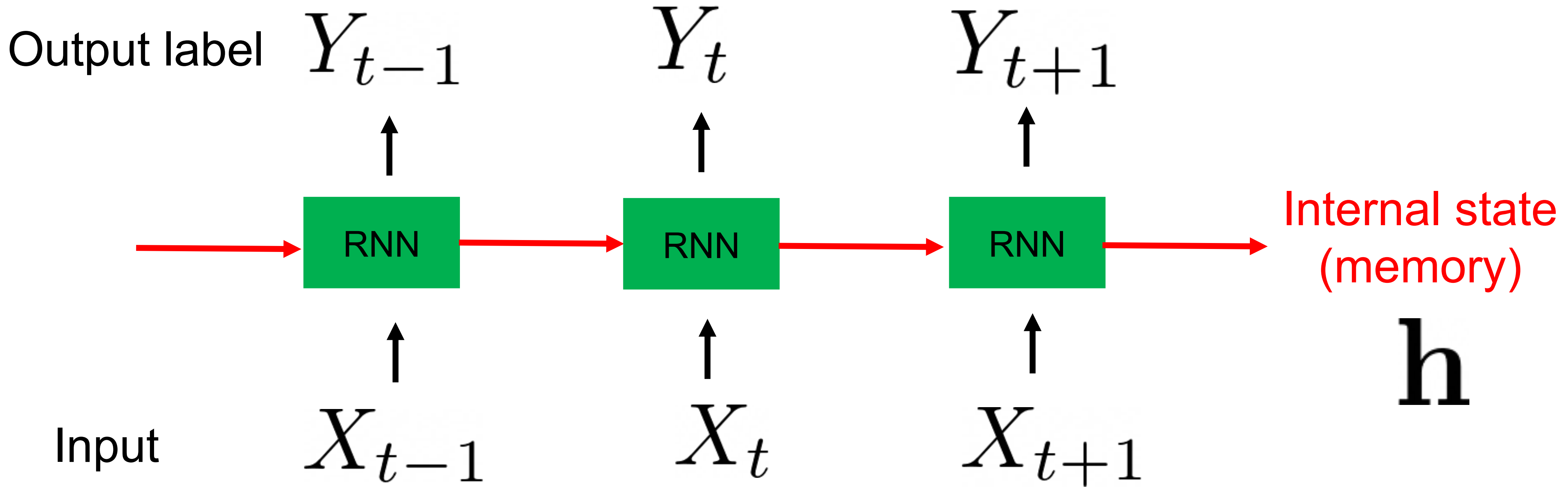
Tag	Meaning	English Examples
ADJ	adjective	*new, good, high, special, big, local*
ADP	adposition	*on, of, at, with, by, into, under*
ADV	adverb	*really, already, still, early, now*
CONJ	conjunction	*and, or, but, if, while, although*
DET	determiner, article	*the, a, some, most, every, no, which*
NOUN	noun	*year, home, costs, time, Africa*
NUM	numeral	*twenty-four, fourth, 1991, 14:24*
PRT	particle	*at, on, out, over per, that, up, with*
PRON	pronoun	*he, their, her, its, my, I, us*
VERB	verb	*is, say, told, given, playing, would*
.	punctuation marks	*. , ; !*
X	other	*ersatz, esprit, dunno, gr8, univeristy*

Sequential Data Labeling

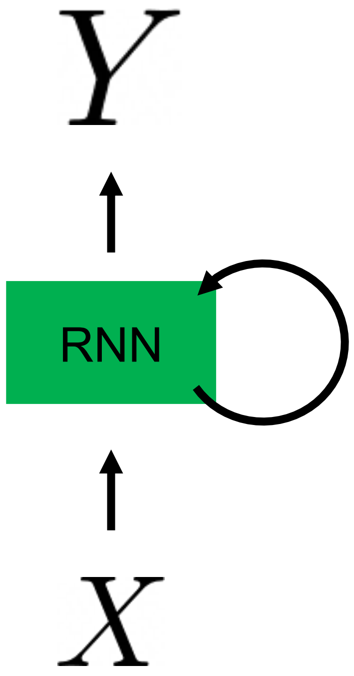


How to capture information across time?

Recurrent Neural Networks



Hidden State Update



Updating function with parameters W

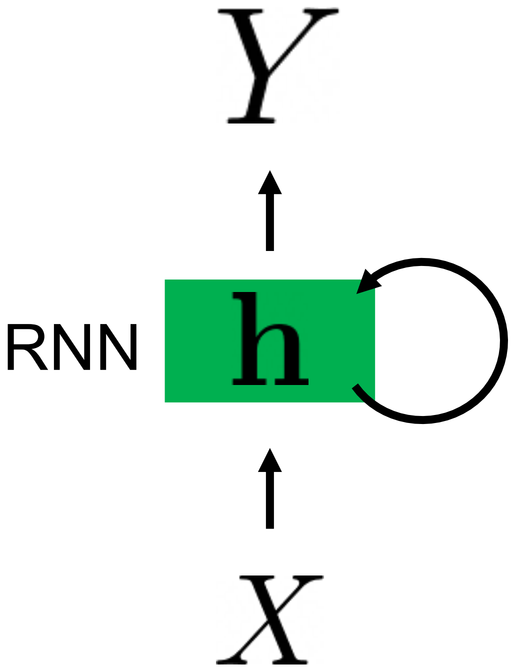
$$\mathbf{h}_t = f_W(\mathbf{h}_{t-1}, \mathbf{x}_t)$$

Hidden state at time t

Hidden state at time t-1

Input at time t

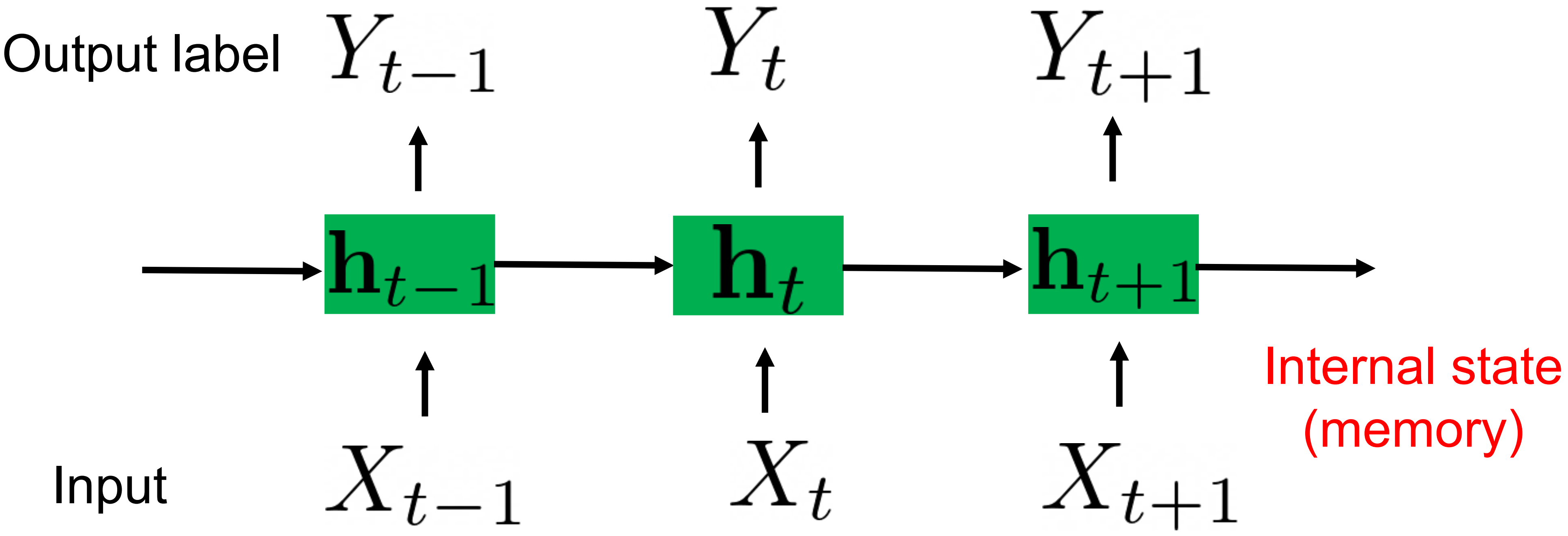
Using the Hidden State



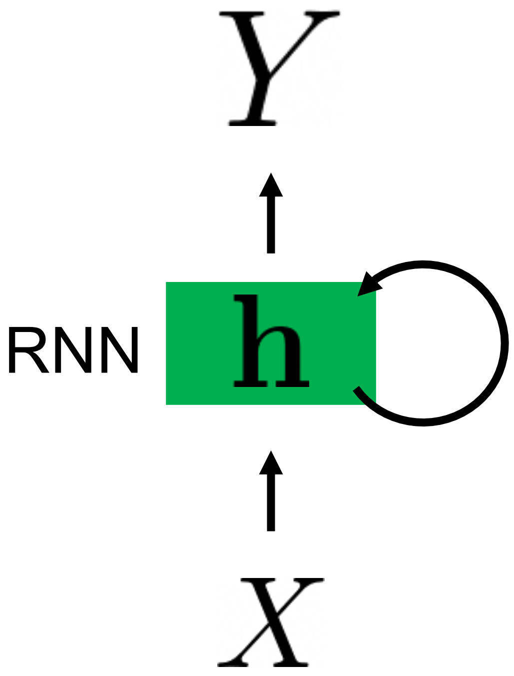
$$\mathbf{h}_t = f_W(\mathbf{h}_{t-1}, \mathbf{x}_t)$$

$$\mathbf{y}_t = f_{W'}(\mathbf{h}_t)$$

Recurrent Neural Networks



Vanilla RNN

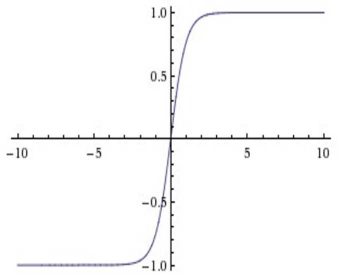


Hidden state updating rule

$$\underbrace{\mathbf{h}_t}_{m \times 1} = \tanh(\underbrace{W_{hh}}_{m \times m}\underbrace{\mathbf{h}_{t-1}}_{m \times 1} + \underbrace{W_{xh}}_{m \times n}\underbrace{\mathbf{x}_t}_{n \times 1})$$

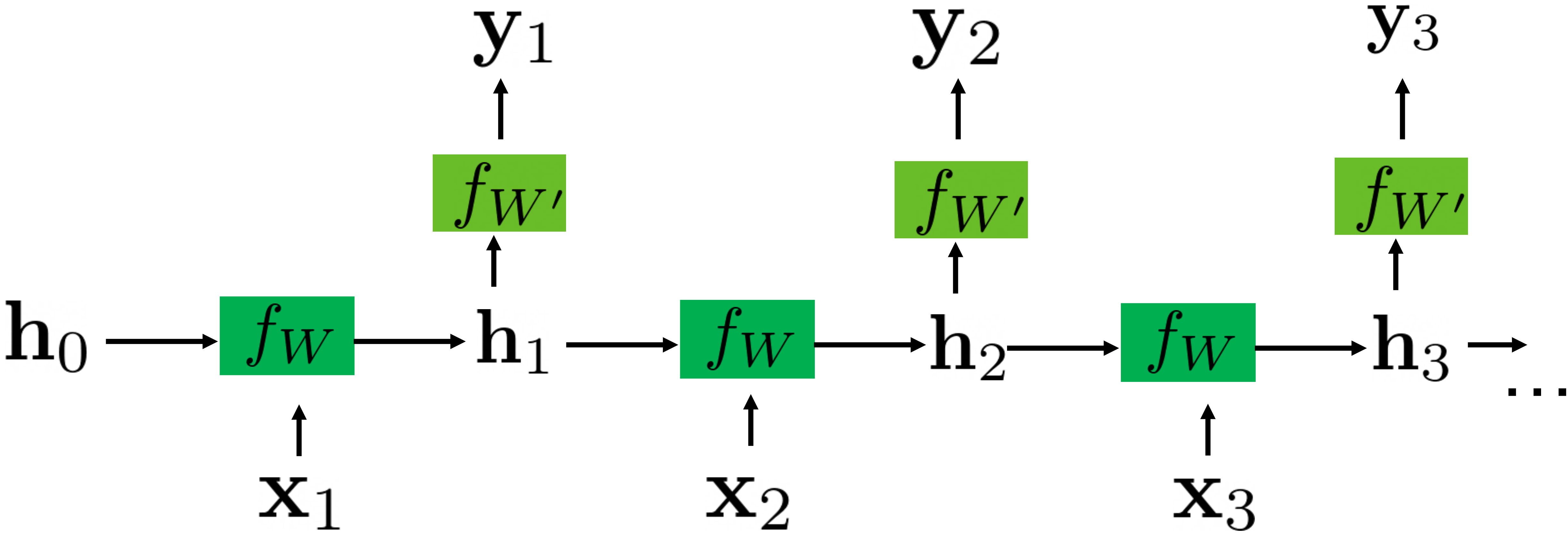
tanh tanh(x)

$$\frac{e^{2x} - 1}{e^{2x} + 1}$$



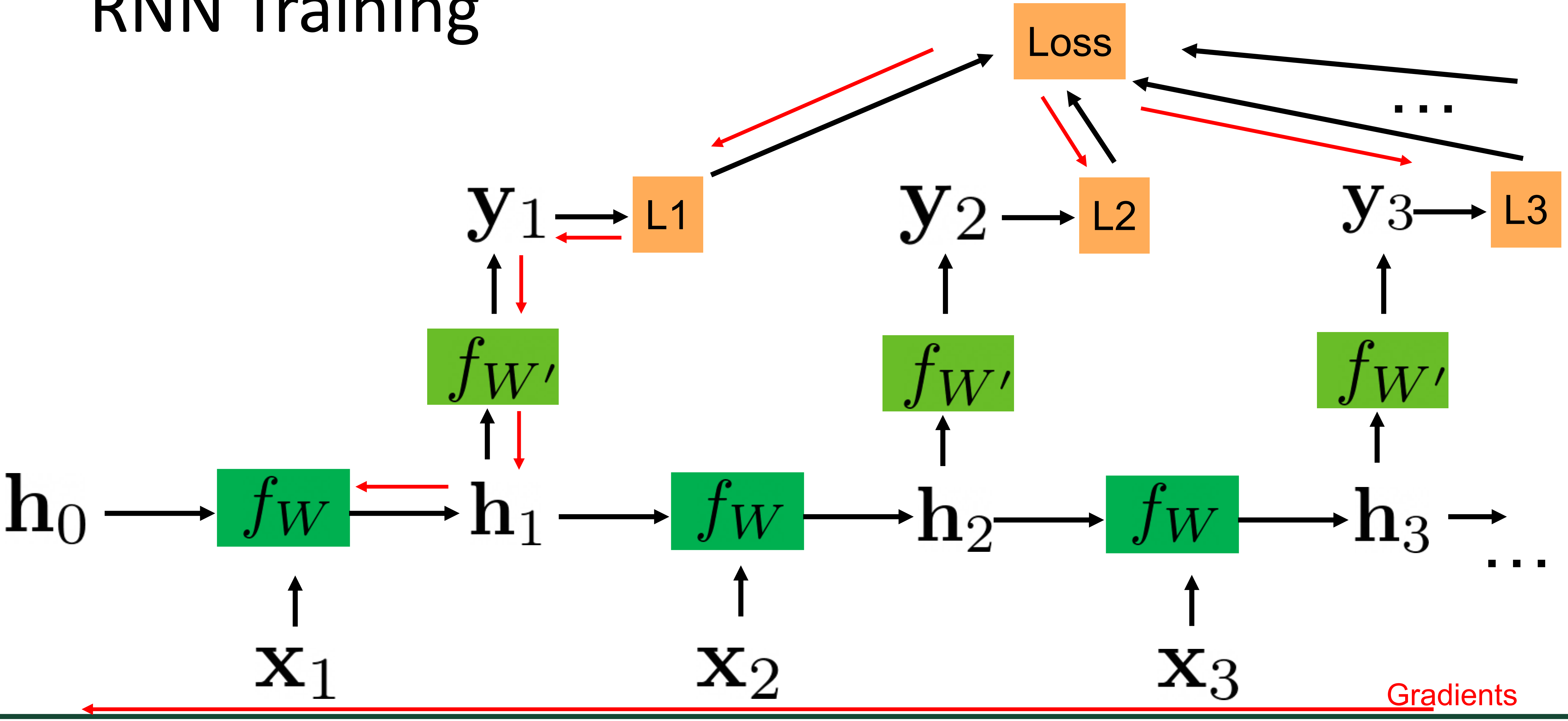
$$\underbrace{\mathbf{y}_t}_{l \times 1} = W_{hy}\underbrace{\mathbf{h}_t}_{m \times 1}$$

RNN Computation Graph

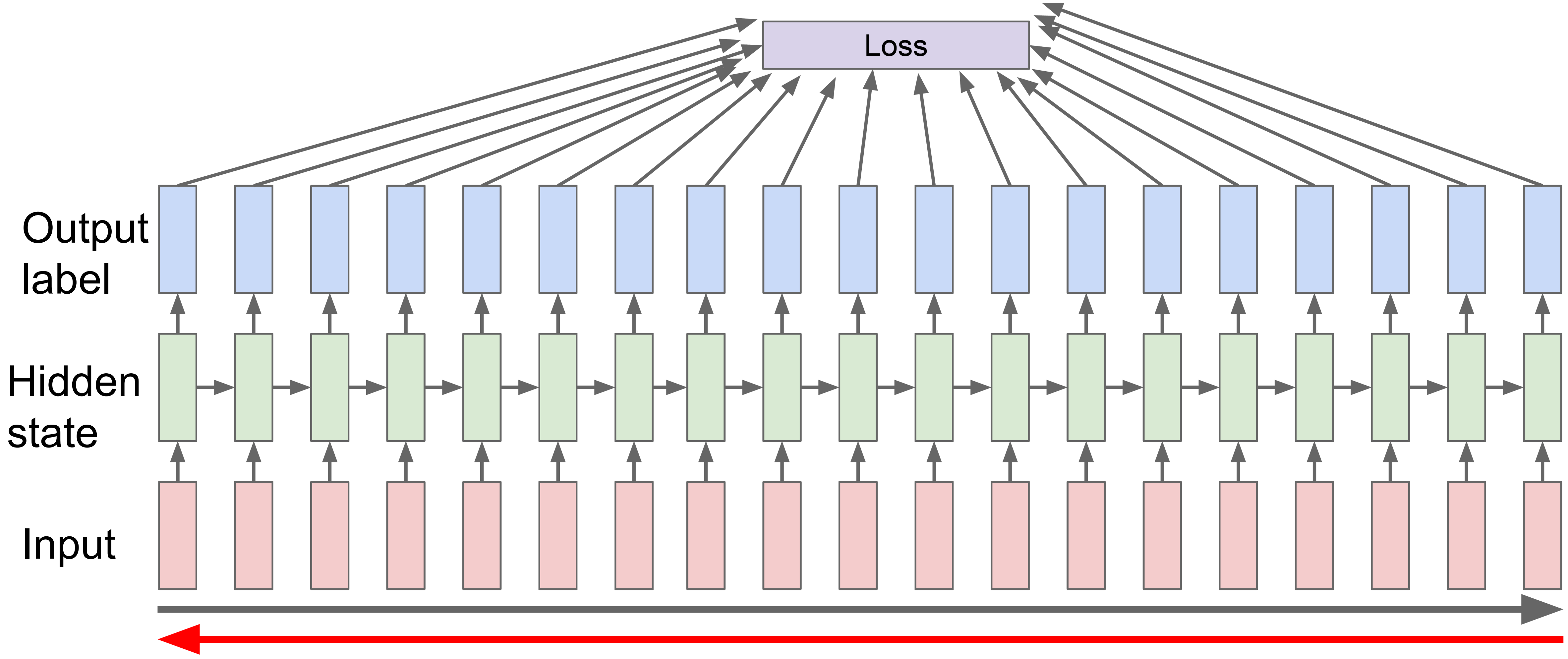


The same set of weights for different time steps f_W $f_{W'}$

RNN Training

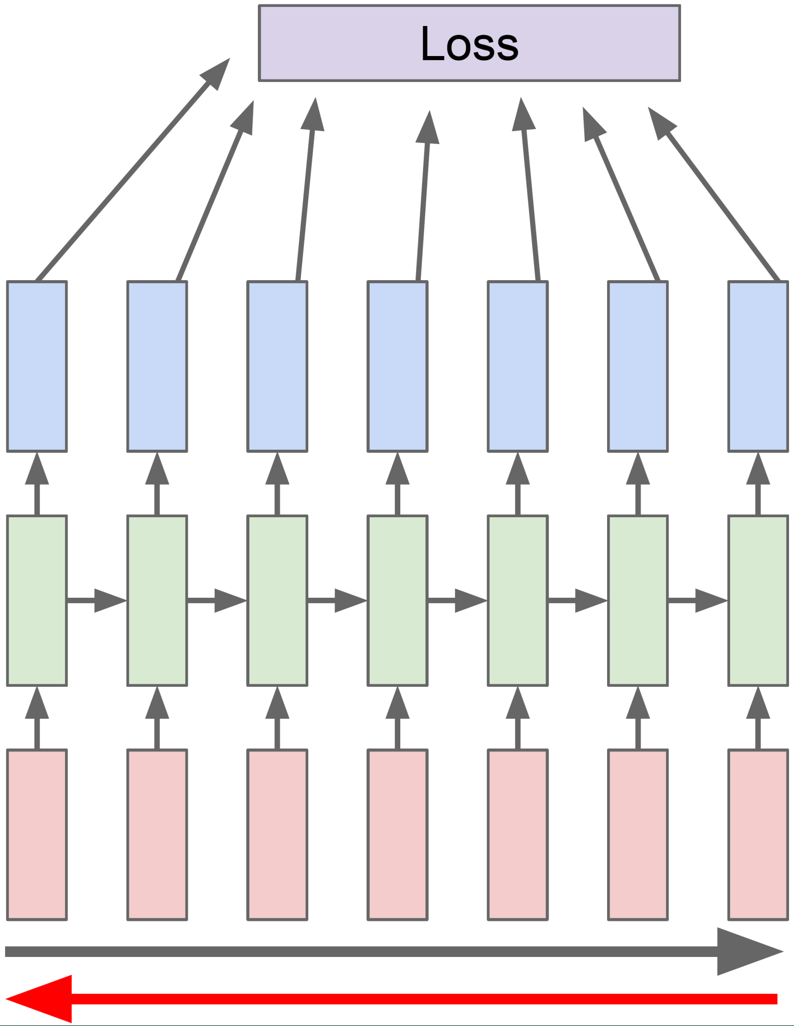


Backpropagation through Time



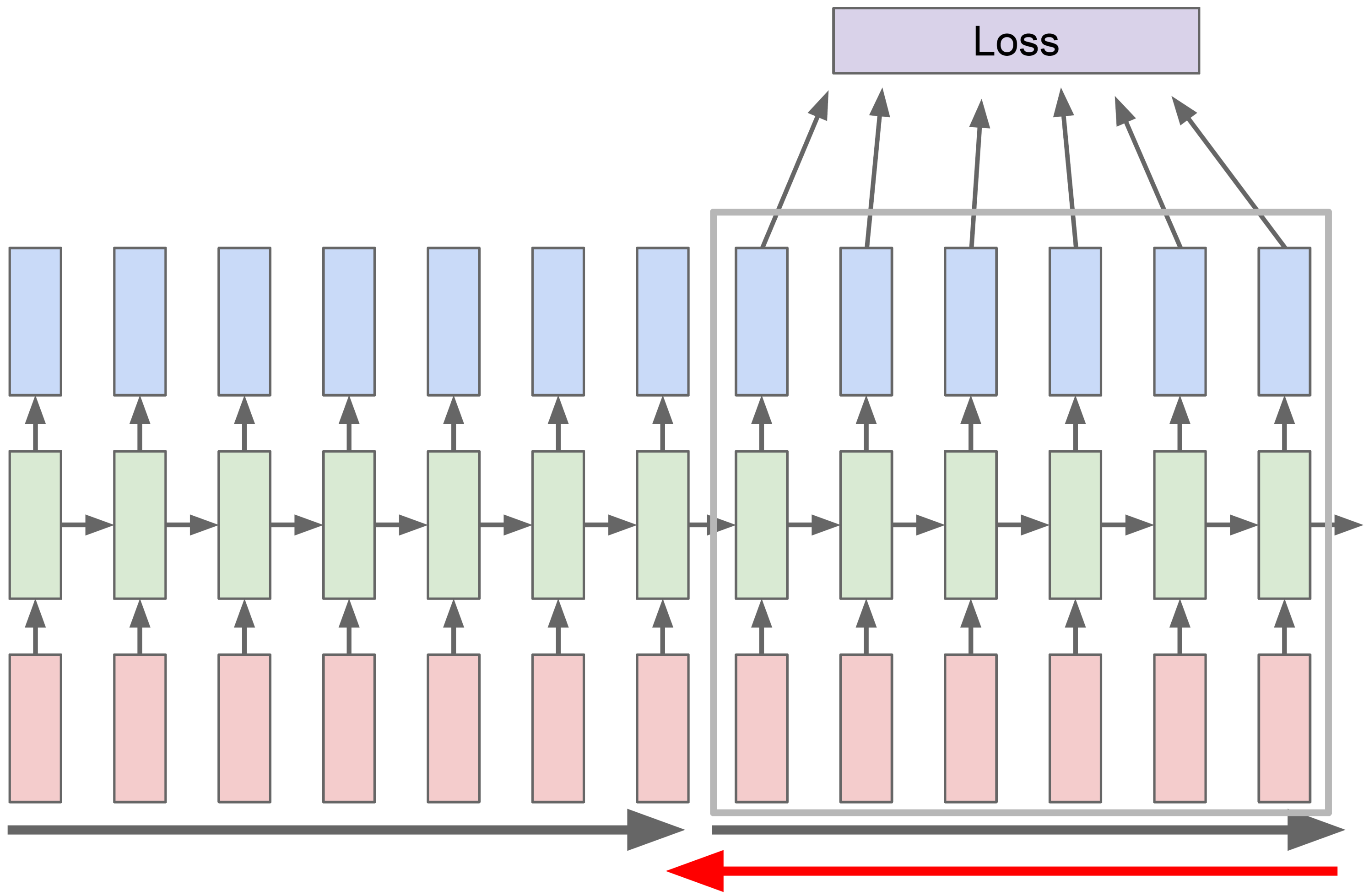
What is the problem in this training paradigm?

Truncated Backpropagation through Time



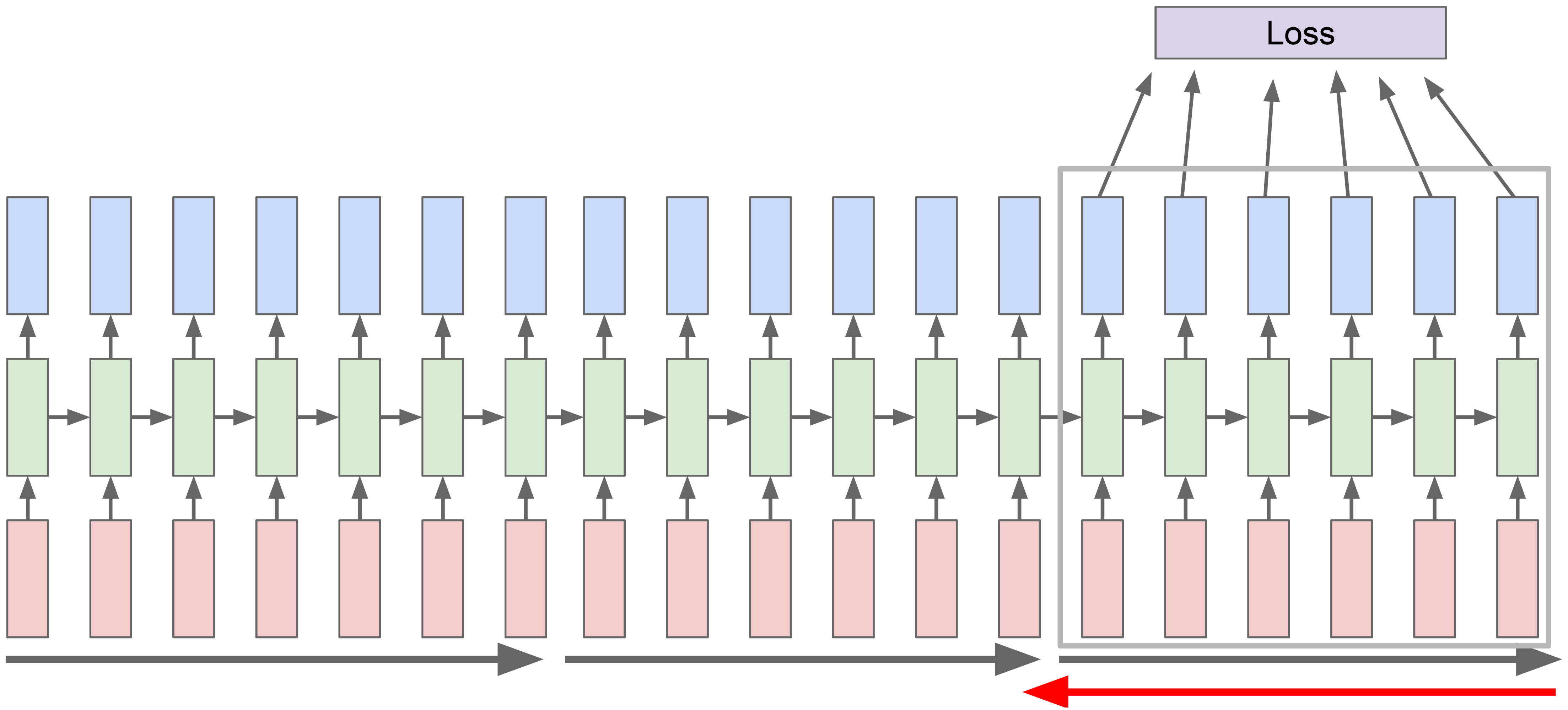
Run forward and backward through chunks of the sequence instead of whole sequence

Truncated Backpropagation through Time

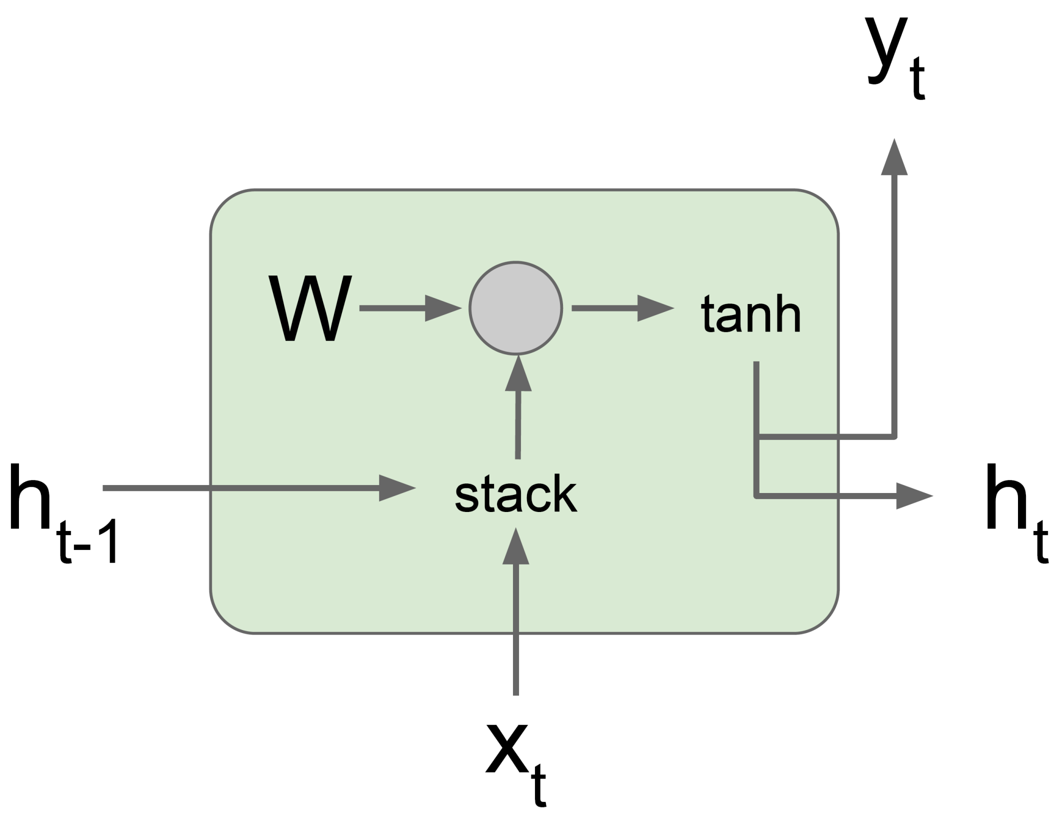


Carry hidden states forward in time forever, but only backpropagate for some smaller number of steps

Truncated Backpropagation through Time

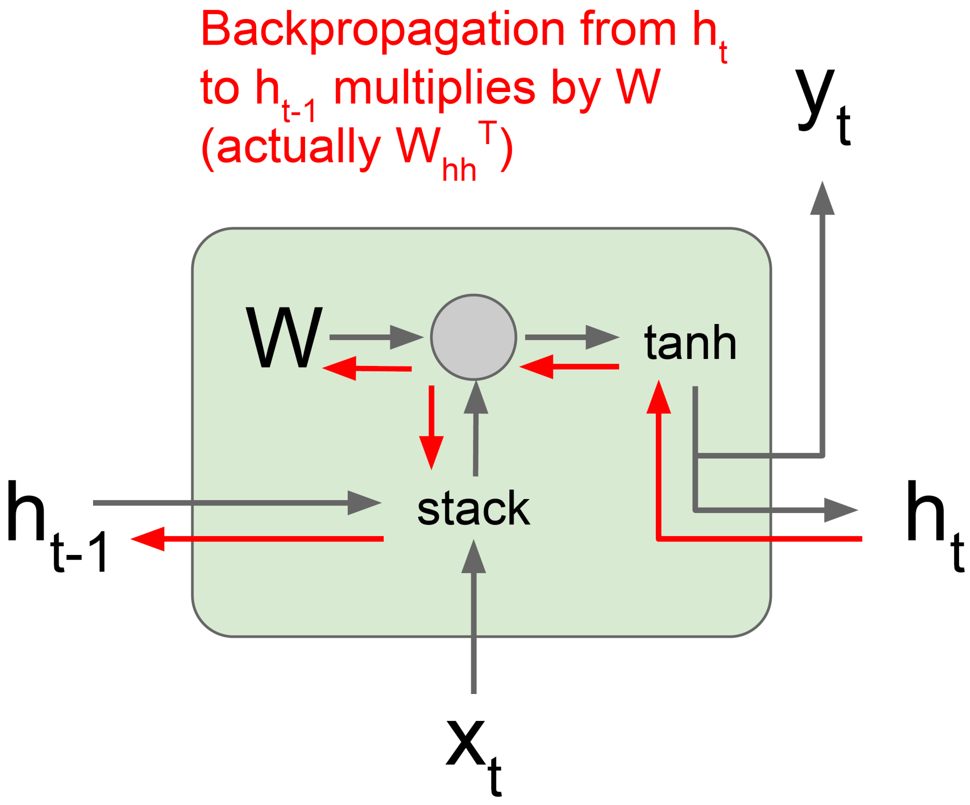


Vanilla RNN Gradient Flow



$$
\begin{aligned}
\mathbf{h}_t &= \tanh(W_{hh}\mathbf{h}_{t-1} + W_{xh}\mathbf{x}_t) \\
&= \tanh\left(\begin{pmatrix} W_{hh} & W_{hx} \end{pmatrix} \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right) \\
&= \tanh\left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)
\end{aligned}
$$

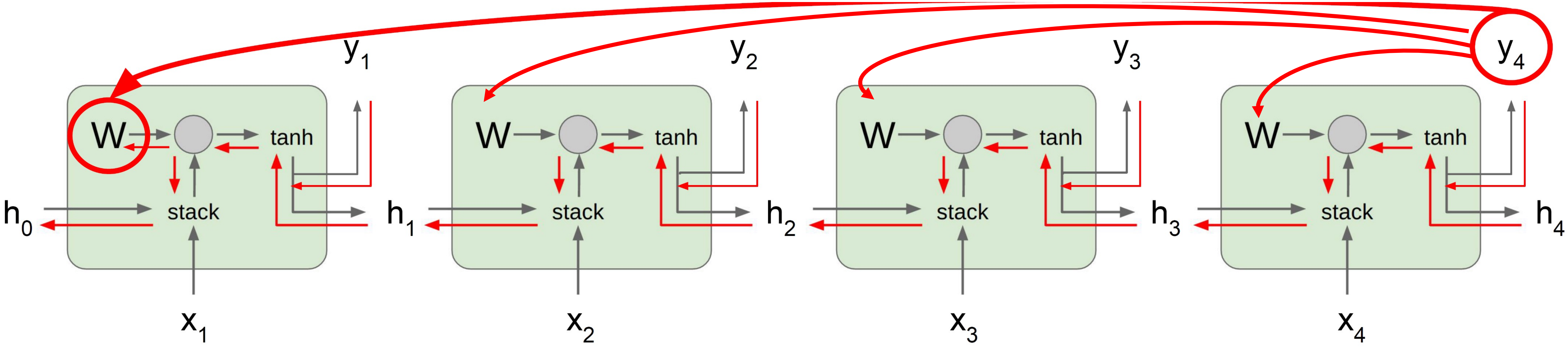
Vanilla RNN Gradient Flow



$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$
$$= \tanh\left(\begin{pmatrix} W_{hh} & W_{hx} \end{pmatrix} \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)$$
$$= \tanh\left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)$$

$$\frac{\partial h_t}{\partial h_{t-1}} = tanh'(W_{hh}h_{t-1} + W_{xh}x_t)W_{hh}$$

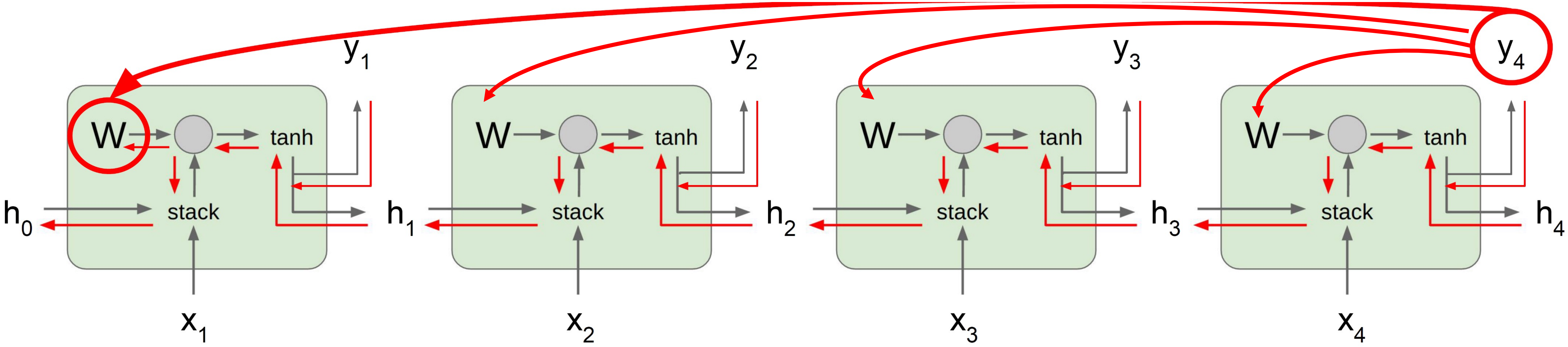
Vanilla RNN Gradient Flow



$$\frac{\partial L}{\partial W} = \sum_{t=1}^{T} \frac{\partial L_t}{\partial W}$$

$$\frac{\partial L_T}{\partial W} = \frac{\partial L_T}{\partial h_T} \frac{\partial h_t}{\partial h_{t-1}} \cdots \frac{\partial h_1}{\partial W} = \frac{\partial L_T}{\partial h_T} \left(\prod_{t=2}^{T} \frac{\partial h_t}{\partial h_{t-1}} \right) \frac{\partial h_1}{\partial W}$$

Vanilla RNN Gradient Flow



$$\frac{\partial L_T}{\partial W} = \frac{\partial L_T}{\partial h_T}\left(\prod_{t=2}^{T} \frac{\partial h_t}{\partial h_{t-1}}\right)\frac{\partial h_1}{\partial W}$$

https://en.wikipedia.org/wiki/Matrix_norm

- Vanishing gradients $\left\|\frac{\partial h_t}{\partial h_{t-1}}\right\|_2 < 1$
- Exploding gradients $\left\|\frac{\partial h_t}{\partial h_{t-1}}\right\|_2 > 1$

Vanilla RNN Gradient Flow

Exploding gradients $$\|\frac{\partial h_t}{\partial h_{t-1}}\|_2 > 1$$

- Gradient clipping

```
grad_norm = np.sum(grad * grad)
if grad_norm > threshold:
  grad *= (threshold / grad_norm)
```

Vanishing gradients $$\|\frac{\partial h_t}{\partial h_{t-1}}\|_2 < 1$$

- Change RNN architecture

Long Short Term Memory (LSTM)

Vanilla RNN

$$h_t = \tanh\left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)$$

LSTM

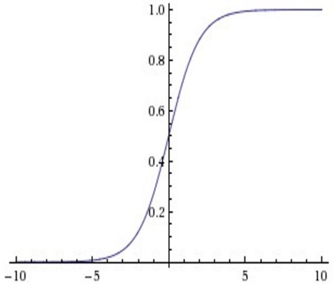
$$\begin{array}{l} \text{Input gate} \\ \text{forget gate} \\ \text{output gate} \\ \text{update} \end{array} \begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

Cell $c_t = f \odot c_{t-1} + i \odot g$

Hidden state $h_t = o \odot \tanh(c_t)$

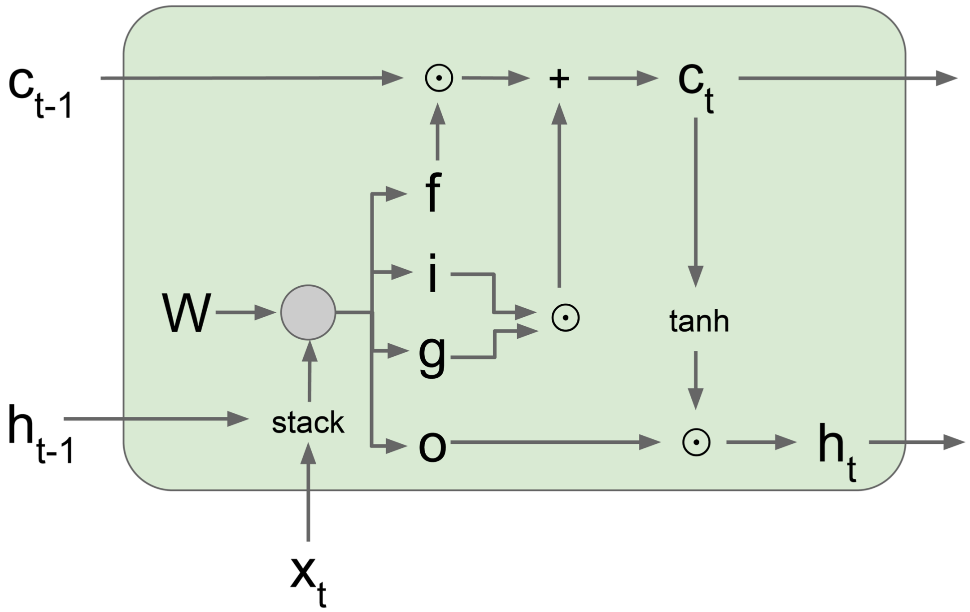
Sigmoid

$$\sigma(x) = 1/(1 + e^{-x})$$



Store Cell and hidden states

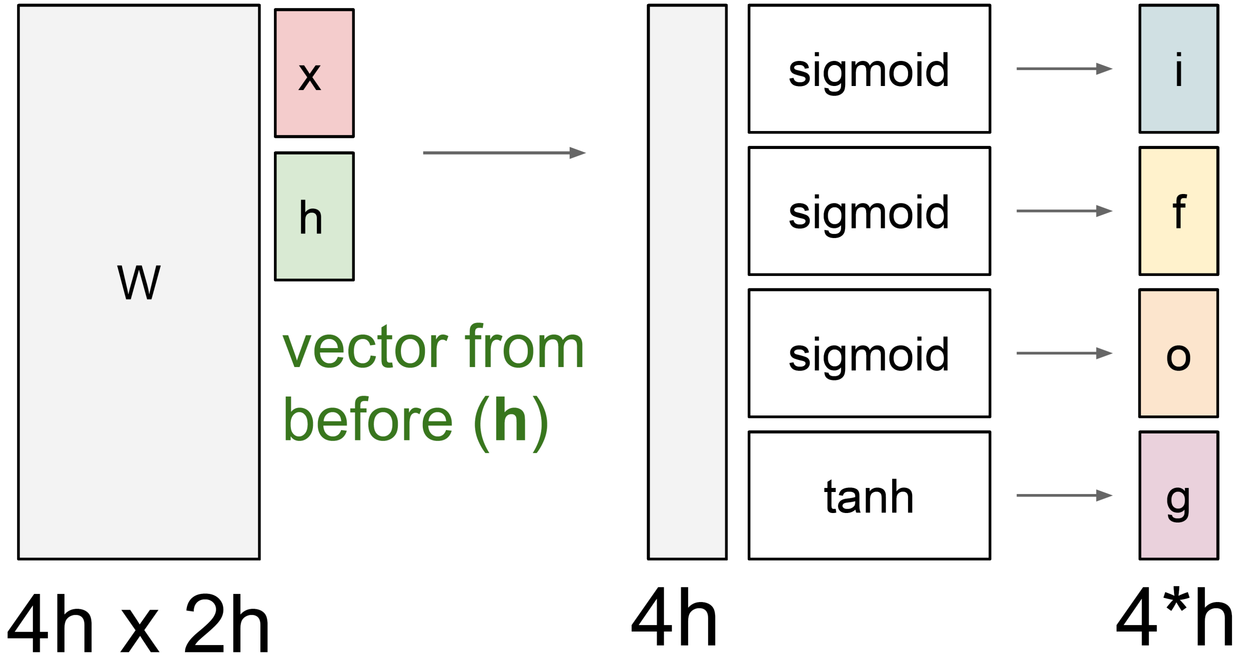
Long Short Term Memory (LSTM)



$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$



- **g**: update, how much to write to cell
- **i**: Input gate, whether to write to cell
- **f**: Forget gate, whether to erase cell
- **o**: Output gate, how much to reveal cell

Long Short Term Memory (LSTM)

Make the RNN easier to preserve information over many steps

- E.g., f = 1 and i = 0
- This is difficult for vanilla RNN

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

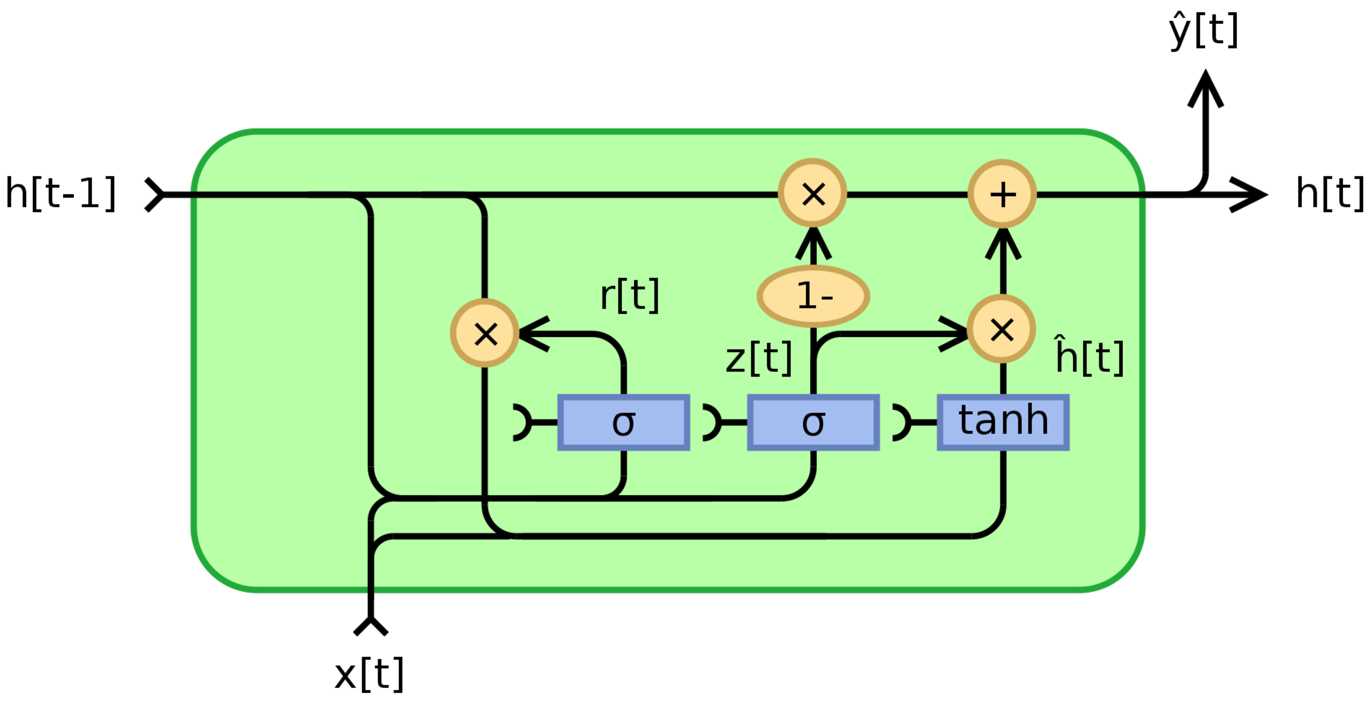
$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

LSTM does not guarantee that there is no vanishing or exploding gradient

It provides an easier way to learn long-distance dependencies

Gated Recurrent Unit (GRU)



$$z_t = \sigma_g(W_z x_t + U_z h_{t-1} + b_z)$$
$$r_t = \sigma_g(W_r x_t + U_r h_{t-1} + b_r)$$
$$\hat{h}_t = \phi_h(W_h x_t + U_h(r_t \odot h_{t-1}) + b_h)$$
$$h_t = (1 - z_t) \odot h_{t-1} + z_t \odot \hat{h}_t$$

- x_t: input vector
- h_t: output vector
- $\hat{h}_t$: candidate activation vector
- z_t: update gate vector
- r_t: reset gate vector
- W, U and b: parameter matrices and vector

https://en.wikipedia.org/wiki/Gated_recurrent_unit

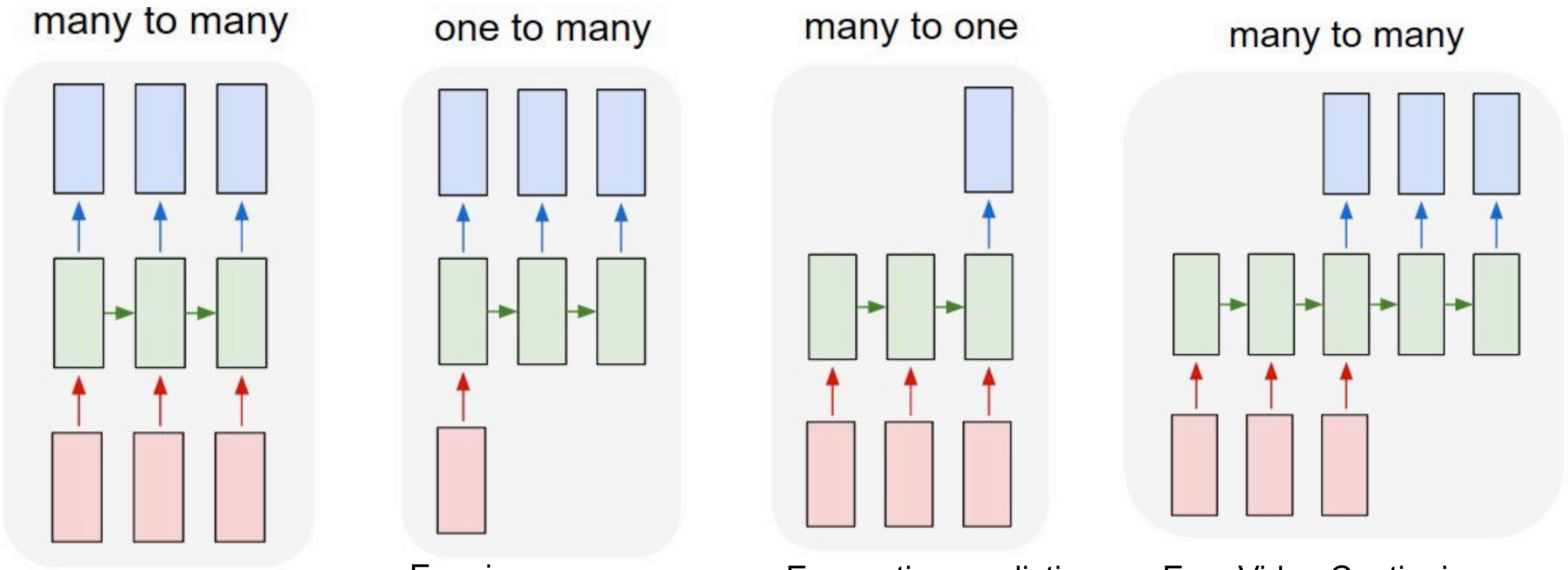
GRUs vs. LSTMs

Both have a forget gate

GRU has fewer parameters, no output gate

GRUs have similar performance compared to LSTMs, have shown better performance on certain datasets

Recurrent Neural Networks



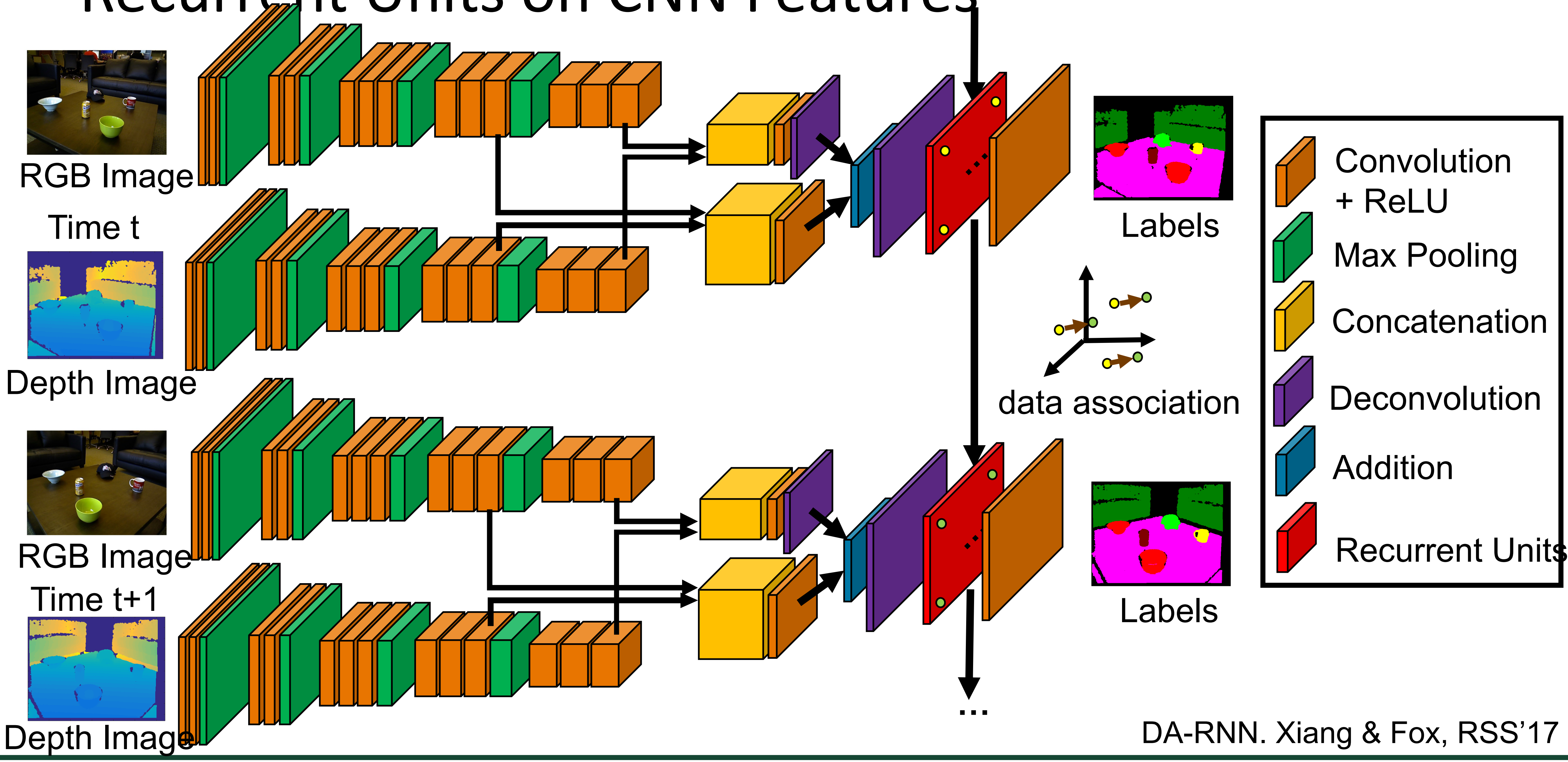
E.g., action recognition on video frames

E.g., image captioning, image -> sequences of words

E.g., action prediction, sequences of frames -> action class

E.g., Video Captioning Sequence of video frames -> caption

Recurrent Units on CNN Features



DA-RNN. Xiang & Fox, RSS'17

Summary

RNNs can be used for sequential data to capture dependencies in time

LSTMs and GRUs are better then vanilla RNNs

It is difficult to capture long-term dependencies in RNNs

Use transformers (next lecture)

Further Reading

Deep Learning Textbook: Sequence Modeling: Recurrent and Recursive Nets
https://www.deeplearningbook.org/contents/rnn.html

Stanford CS231n, lecture 10, Recurrent Neural Networks
http://cs231n.stanford.edu/

Long Short Term Memory
https://www.researchgate.net/publication/13853244_Long_Short-term_Memory
https://colah.github.io/posts/2015-08-Understanding-LSTMs/

Gated Recurrent Units https://arxiv.org/pdf/1412.3555.pdf