Visual Perception: Depth Perception

CS 6334 Virtual Reality

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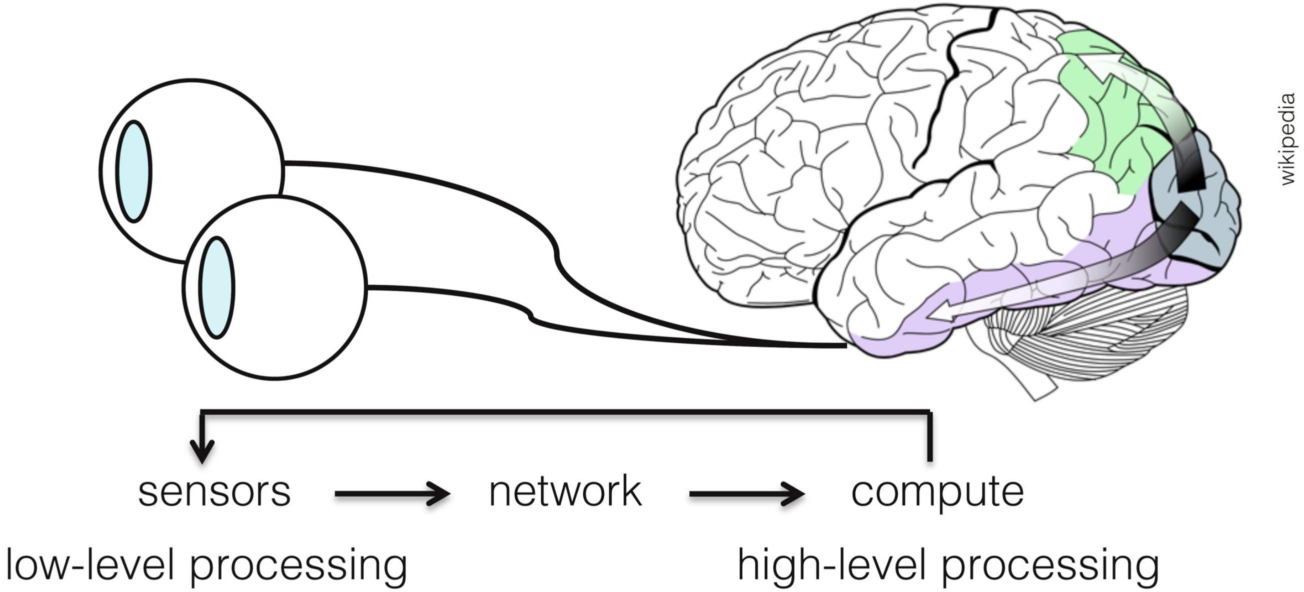
Visual Perception

- How humans perceive or interpret the real world using vision?



- We need to understand visual perception to achieve visual unawareness in VR systems

Depth Perception

- Metric
  - The car is 10 meters away
- Ordinary
  - The tree is behind the car

Depth Cues

- Information for sensory stimulation that is relevant to depth perception

- Monocular cues: single eye

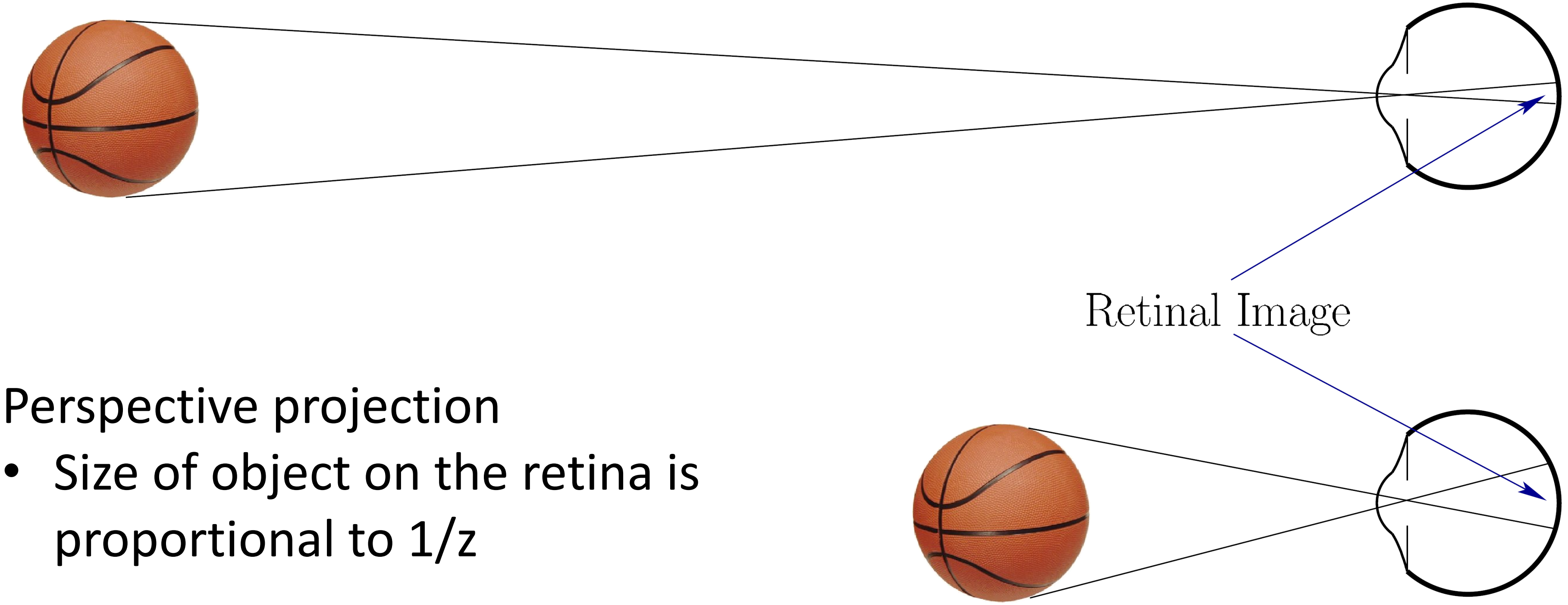
- Stereo cues: both eyes

"Paris Street, Rainy Day," Gustave Caillebotte, 1877. Art Institute of Chicago

- Texture of the bricks
- Perspective projection
- Etc.

Monocular Depth Cues

- Retinal image size

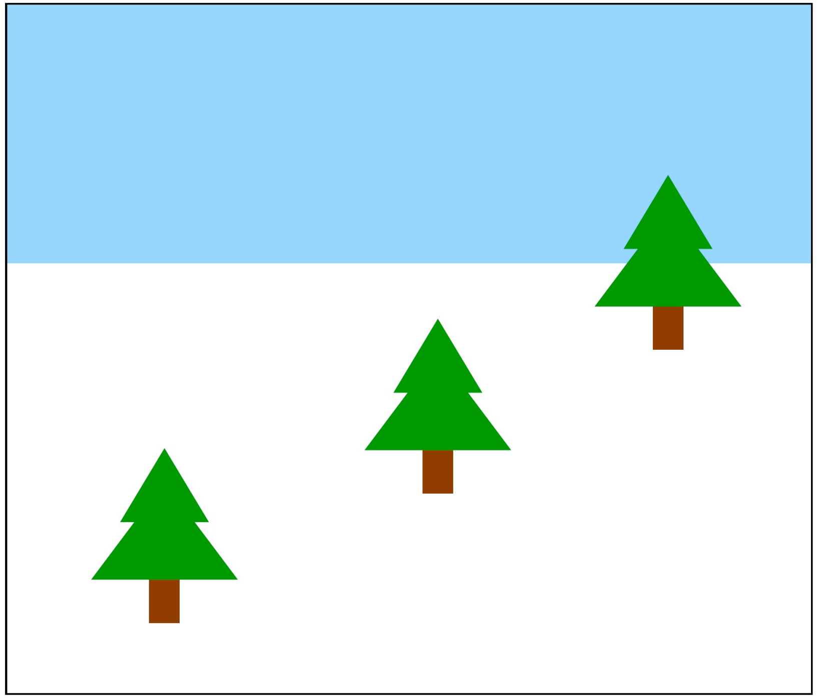


Perspective projection

- Size of object on the retina is proportional to 1/z

Monocular Depth Cues

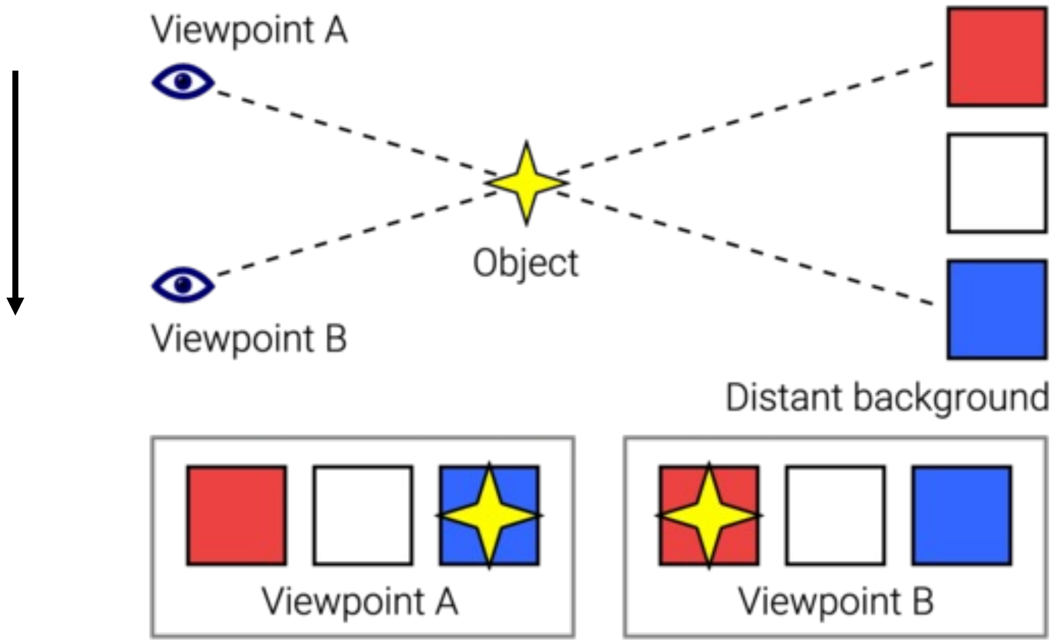
- Height in visual field
  - The closer to the horizon, the further the perceived distance



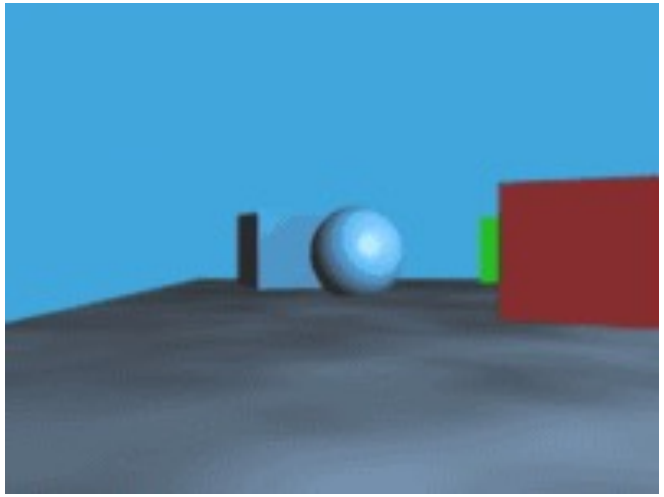
size constancy scaling

Monocular Depth Cues

- Motion parallax
  - Parallax: relative difference in speed

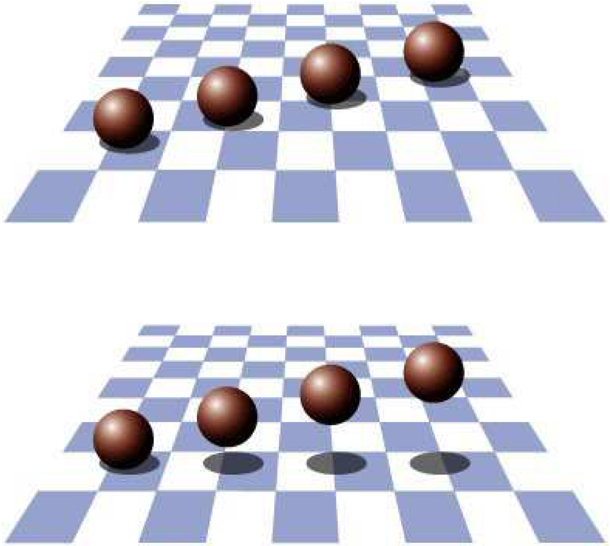


Closer objects have larger image displacements than further objects

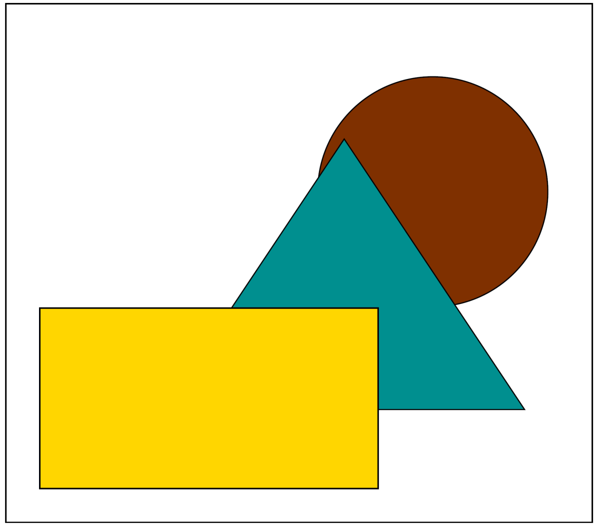


Further objects move slower

Monocular Depth Cues



Shadow



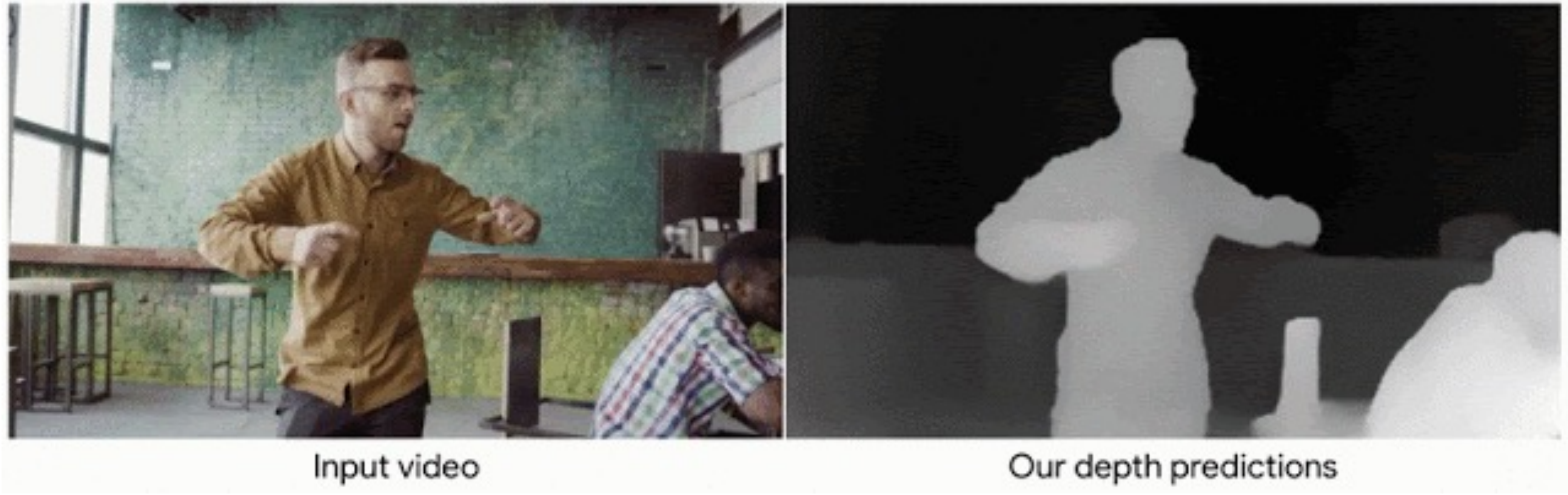
Occlusions

Image blur

Atmospheric cue

further away because it has lower contrast

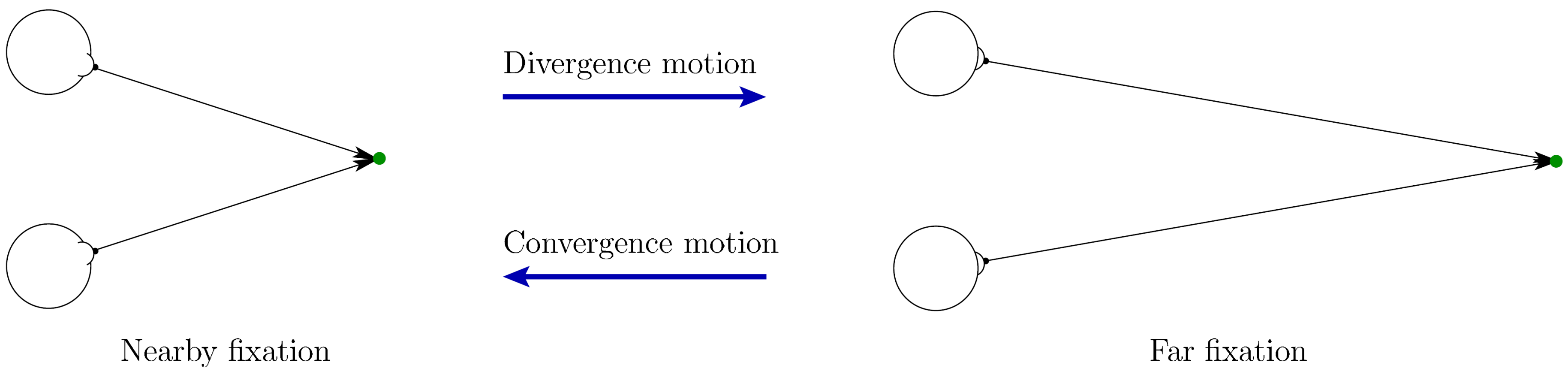
Monocular Depth Estimation



https://heartbeat.fritz.ai/research-guide-for-depth-estimation-with-deep-learning-1a02a439b834

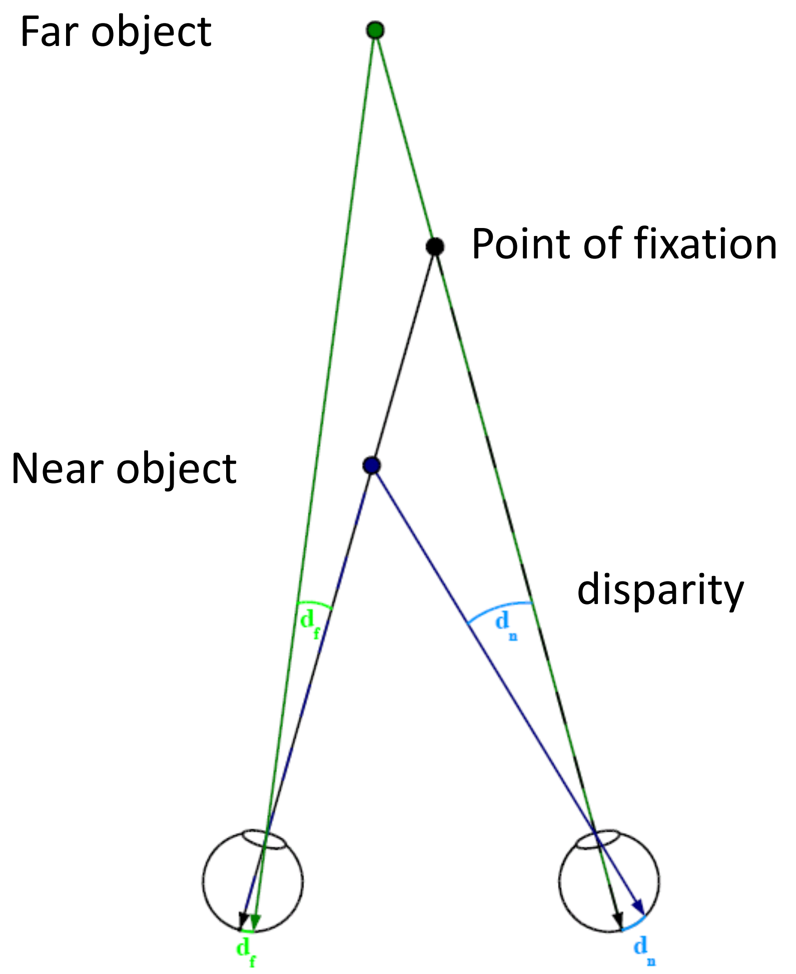
Stereo Depth Cues

- Vergence motion
  - Signals from motor control of the eye muscles



Stereo Depth Cues

- Binocular disparity
  - Each eye provides a different viewpoint, which results in different images on the retina



From Wikipedia

Geometry of Stereo Vision

- Basics: points and lines
- Homogeneous representation of lines

A line in a 2D plane $ax + by + c = 0$ $(a, b, c)^T$

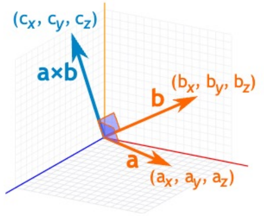
$k(a, b, c)^T$ represents the same line for nonzero k

A point lies on the line $\mathbf{x}^T \mathbf{l} = 0$ $\mathbf{x} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ $\mathbf{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

Points and Lines

When **a** and **b** start at the origin point (0,0,0), the Cross Product will end at:

- $c_x = a_y b_z - a_z b_y$
- $c_y = a_z b_x - a_x b_z$
- $c_z = a_x b_y - a_y b_x$



- Intersection of lines

$$\mathbf{l} = (a, b, c)^T \qquad \mathbf{l}' = (a', b', c')^T$$

Example: The cross product of **a** = (2,3,4) and **b** = (5,6,7)

- $c_x = a_y b_z - a_z b_y = 3\times7 - 4\times6 = -3$
- $c_y = a_z b_x - a_x b_z = 4\times5 - 2\times7 = 6$
- $c_z = a_x b_y - a_y b_x = 2\times6 - 3\times5 = -3$

Answer: **a × b** = (−3,6,−3)

cross product example

The intersection is $\mathbf{x} = \mathbf{l} \times \mathbf{l}'$ (vector cross product)

$$\mathbf{l} \cdot (\mathbf{l} \times \mathbf{l}') = \mathbf{l}' \cdot (\mathbf{l} \times \mathbf{l}') = 0$$

$$\mathbf{l}^T \mathbf{x} = \mathbf{l}'^T \mathbf{x} = 0$$

Points and Lines

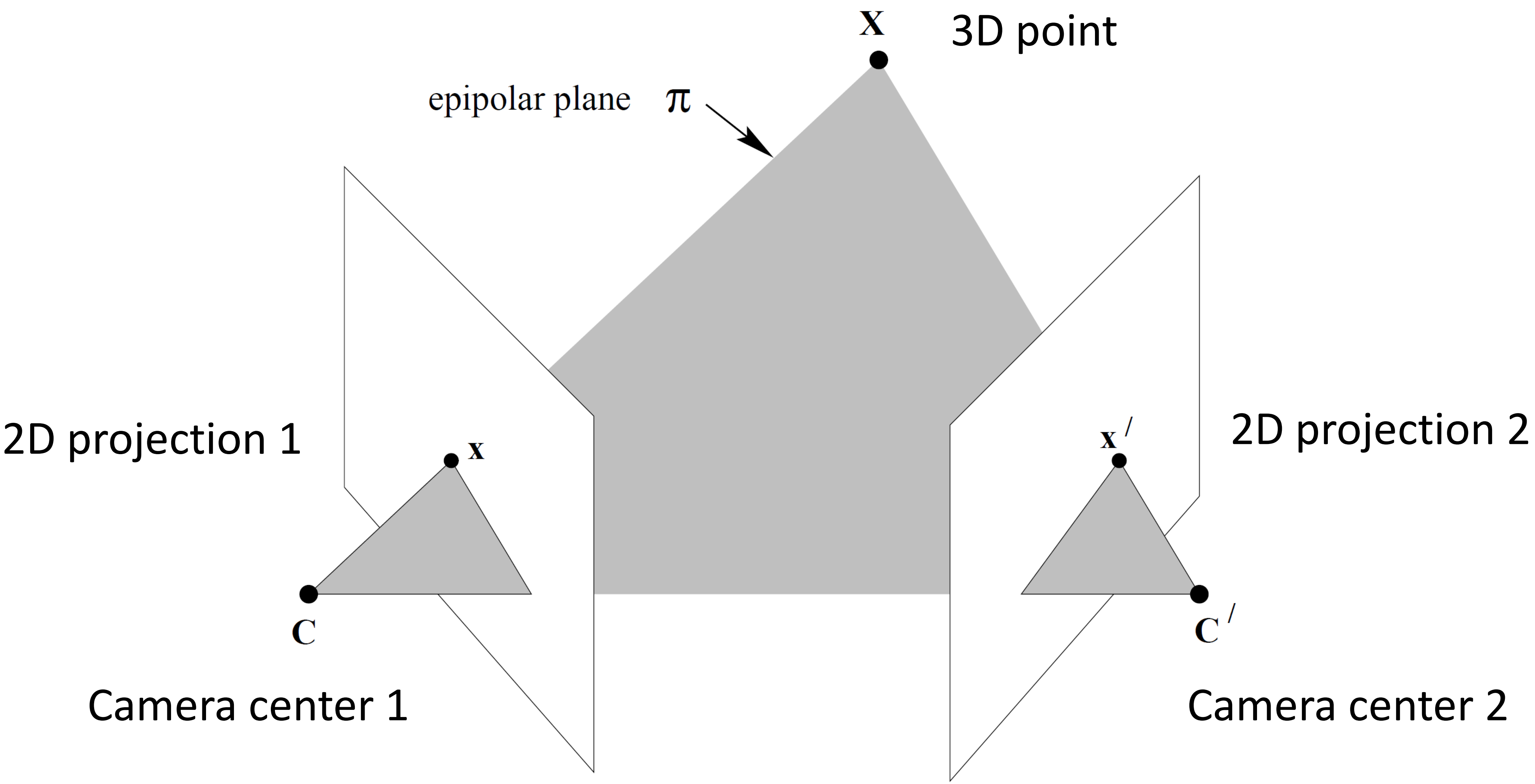
- Line joining points

$$\mathbf{l} = \mathbf{x} \times \mathbf{x}'$$

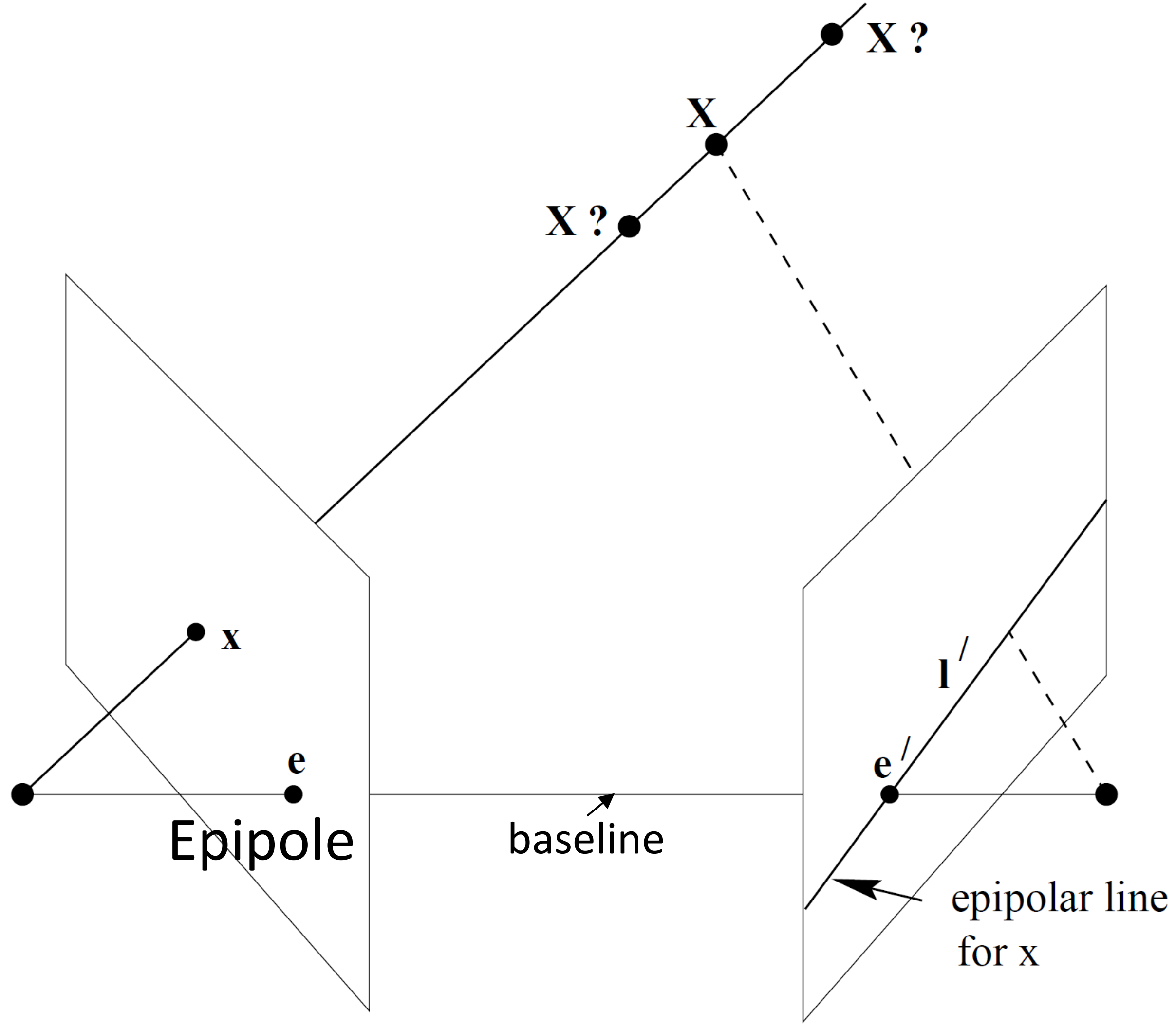
$$\mathbf{x} \cdot (\mathbf{x} \times \mathbf{x}') = \mathbf{x}' \cdot (\mathbf{x} \times \mathbf{x}') = 0$$

$$\mathbf{x}^T \mathbf{l} = \mathbf{x}'^T \mathbf{l} = 0$$

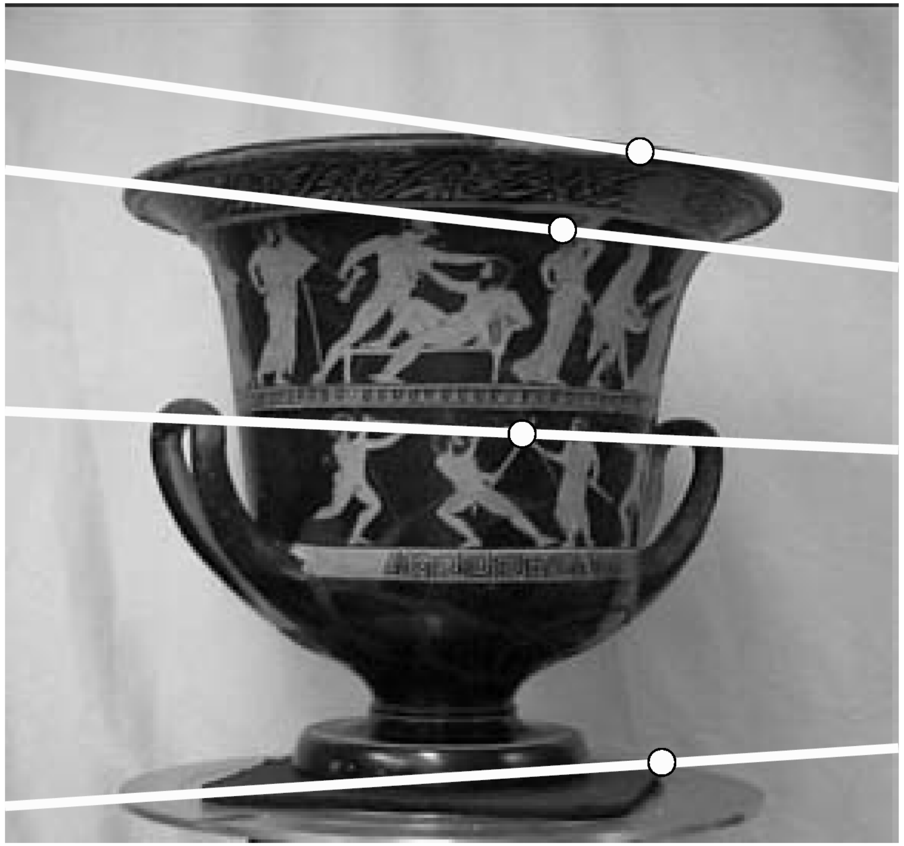
Epipolar Geometry



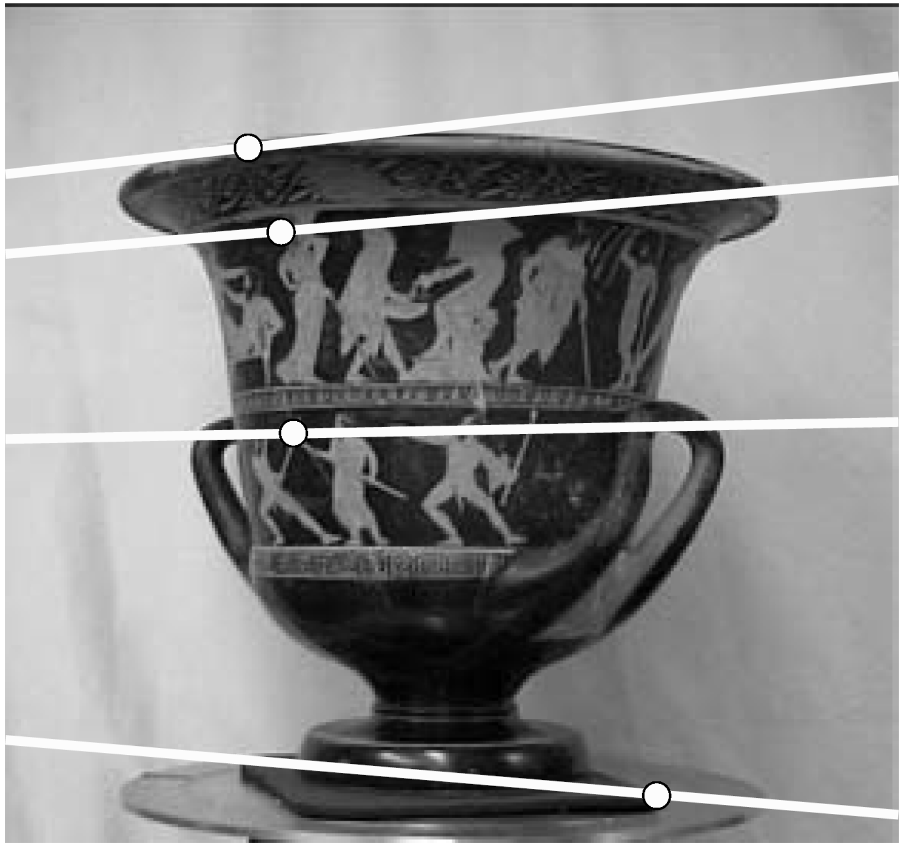
Epipolar Geometry



Epipolar Geometry



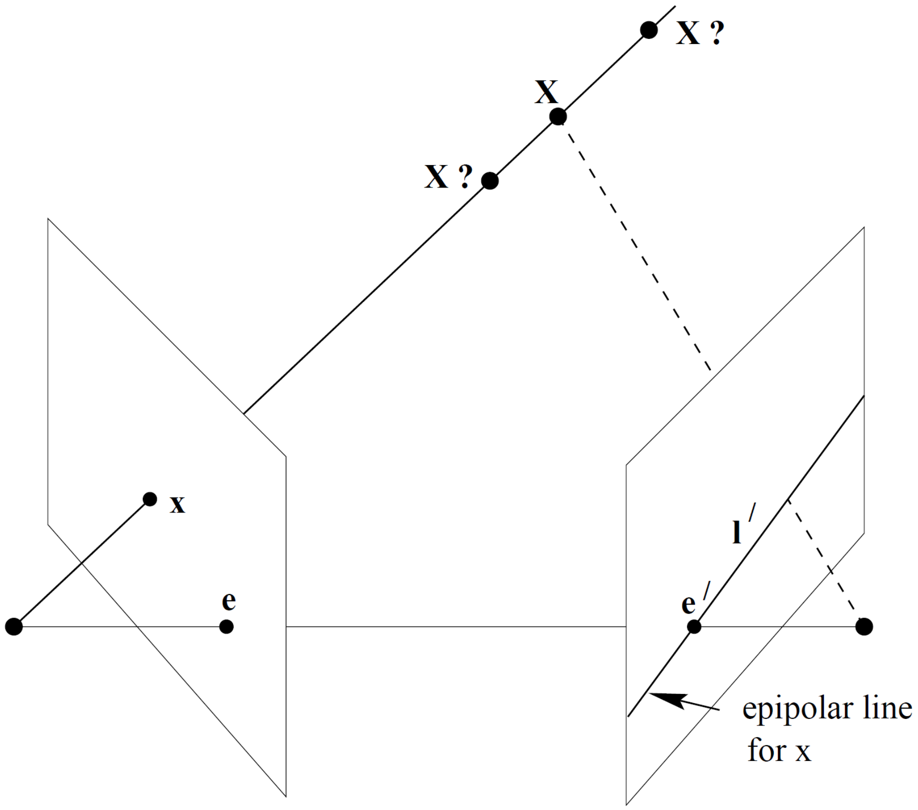
Epipolar lines



Rotation and Translation
between two views

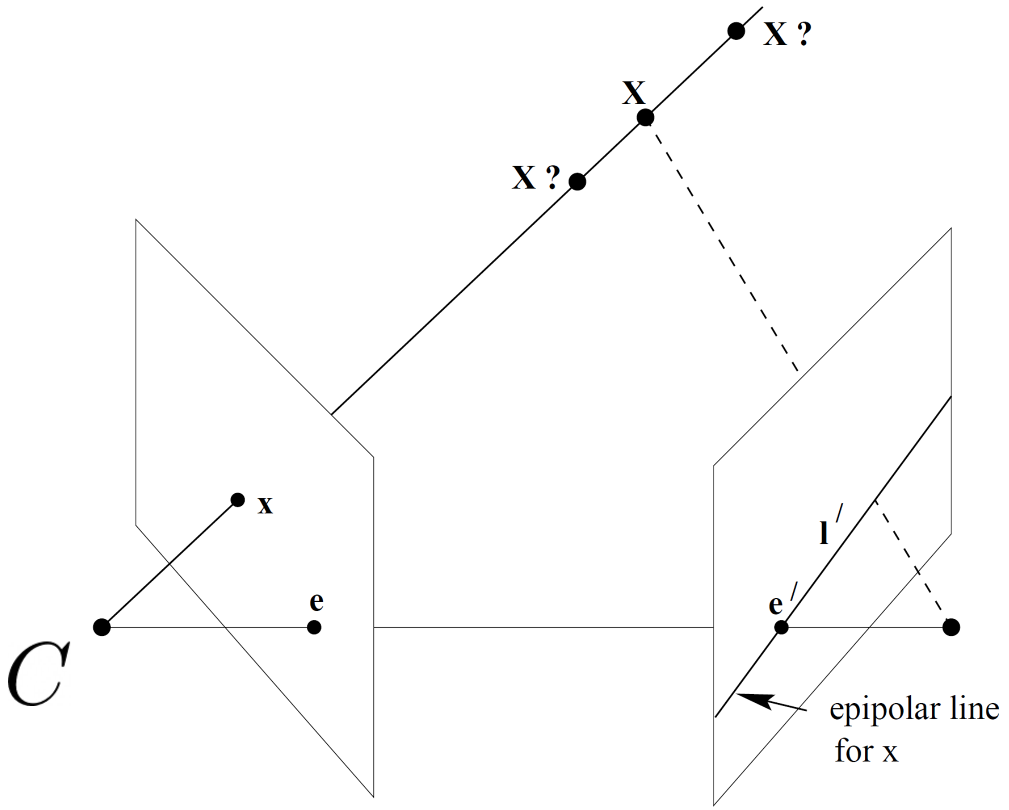
Epipolar Geometry

- What is the mapping for a point in one image to its epipolar line?



$$\mathbf{x} \mapsto \mathbf{l}'$$

Fundamental Matrix



- Recall camera projection

$$P = K[R|\mathbf{t}]$$

$$\mathbf{x} = P\mathbf{X}$$ Homogeneous coordinates

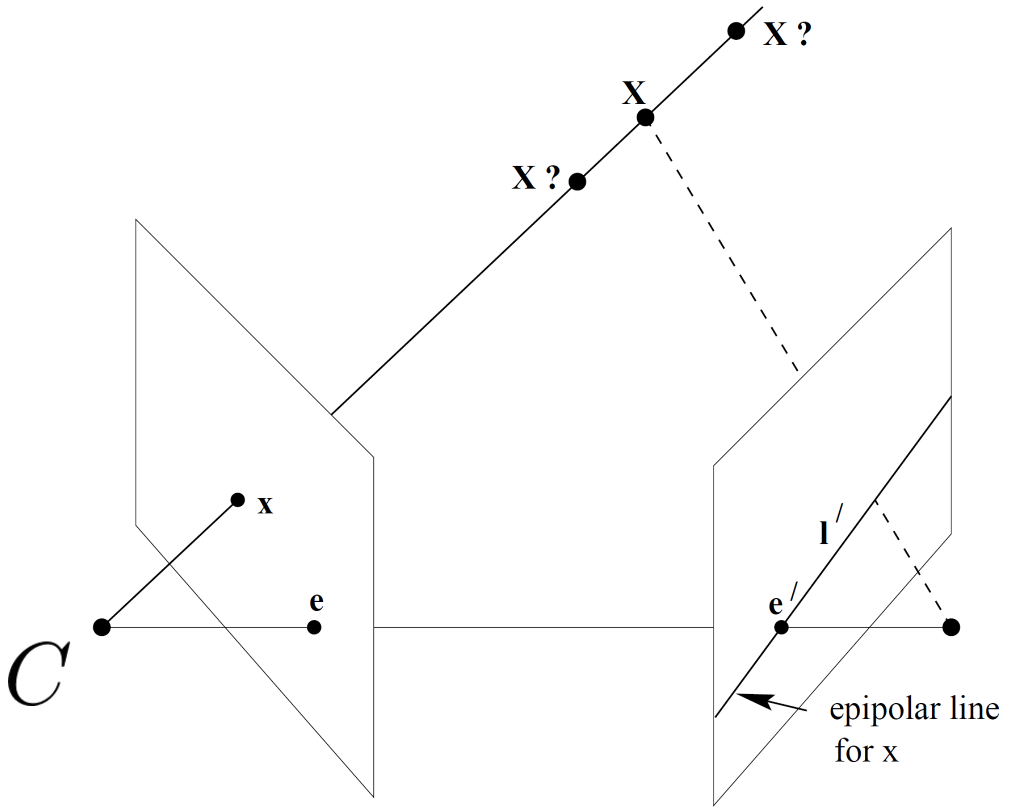
- Backprojection

$$\mathbf{X}(\lambda) = \mathrm{P}^{+}\mathbf{x} + \lambda\mathbf{C}$$

P^+ is the pseudo-inverse of P, $PP^+ = I$

$P^+\mathbf{x}$ and C are two points on the ray

Fundamental Matrix



- Project to the other image

$P^{+}\mathbf{x}$ and C are two points on the ray

$P'P^{+}\mathbf{x}$ and $P'C$

- Epipolar line

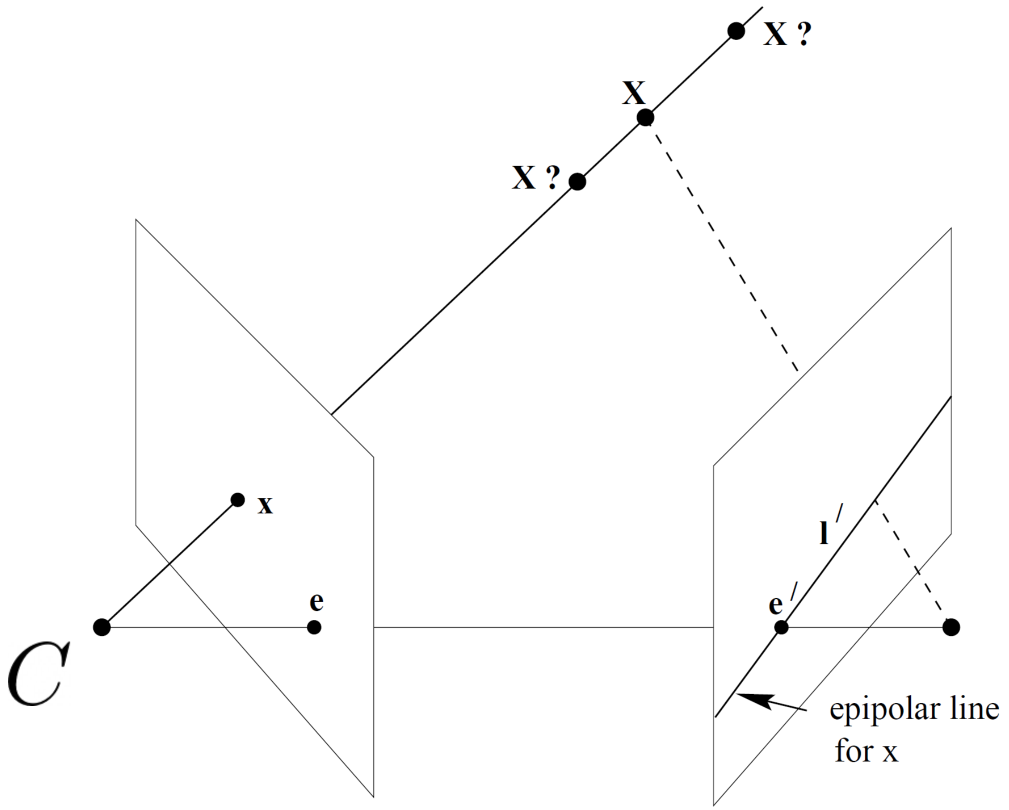
$$\mathbf{l}' = (P'C) \times (P'P^{+}\mathbf{x})$$

Epipole $\mathbf{e}' = (P'C)$

$$\mathbf{l}' = [\mathbf{e}']_{\times}(P'P^{+}\mathbf{x})$$

Cross product matrix

Fundamental Matrix



- Epipolar line

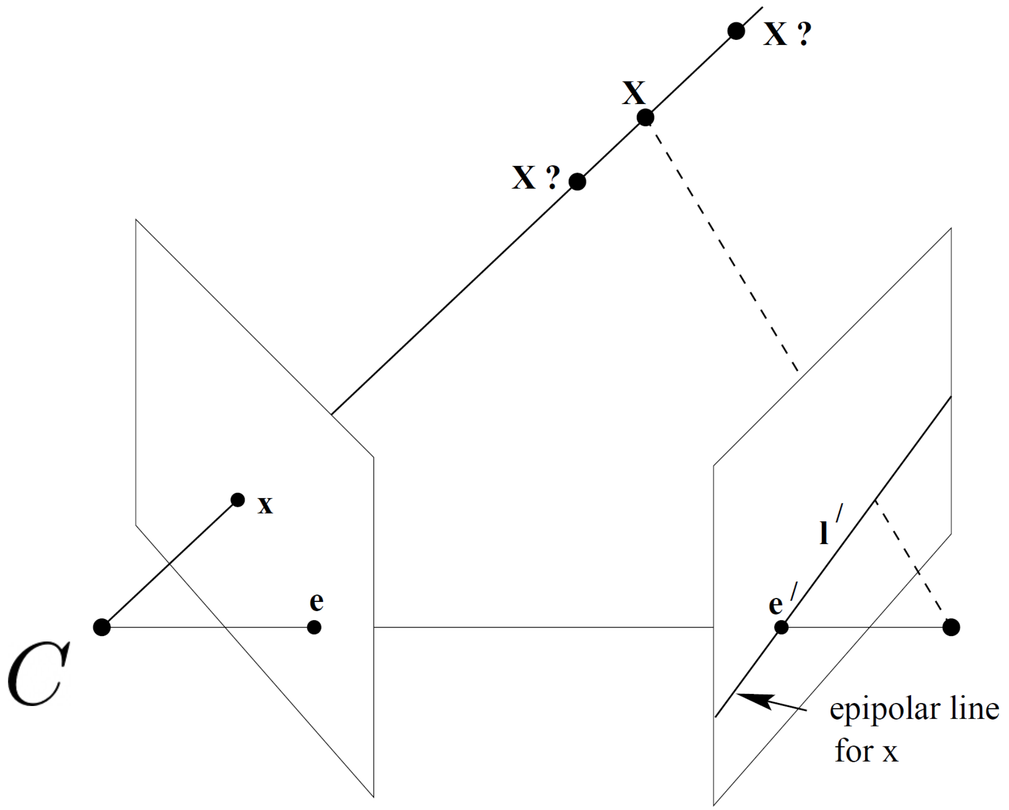
$$\mathbf{l}' = [\mathbf{e}']_\times (P' P^+ \mathbf{x}) = F\mathbf{x}$$

- Fundamental matrix

$$F = [\mathbf{e}']_\times P' P^+$$

3x3

Properties of Fundamental Matrix



$\mathbf{x}'$ is on the epiploar line $\mathbf{l}' = F\mathbf{x}$

$$\mathbf{x}'^T F \mathbf{x} = 0$$

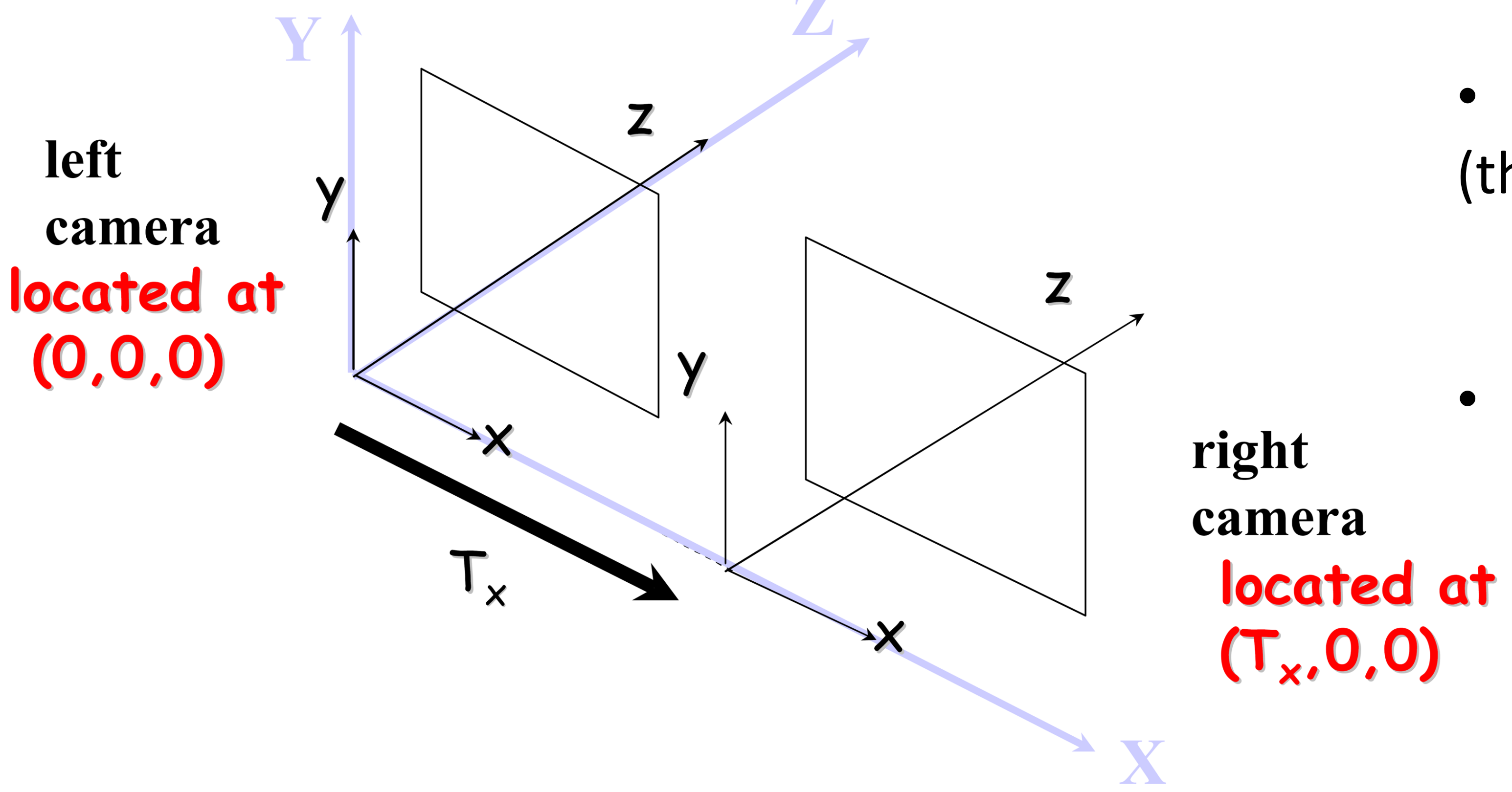
- Transpose: if F is the fundamental matrix of (P, P'), then F^T is the fundamental matrix of (P', P)
- Epipolar line: $\mathbf{l}' = F\mathbf{x}$ $\quad \mathbf{l} = F^T\mathbf{x}'$
- Epipole: $\mathbf{e}'^T \mathbf{F} = \mathbf{0}$ $\quad \mathbf{F}\mathbf{e} = \mathbf{0}$

$$\mathbf{e}'^T(\mathbf{F}\mathbf{x}) = (\mathbf{e}'^T\mathbf{F})\mathbf{x} = 0 \text{ for all } \mathbf{x}$$

- 7 degrees of freedom

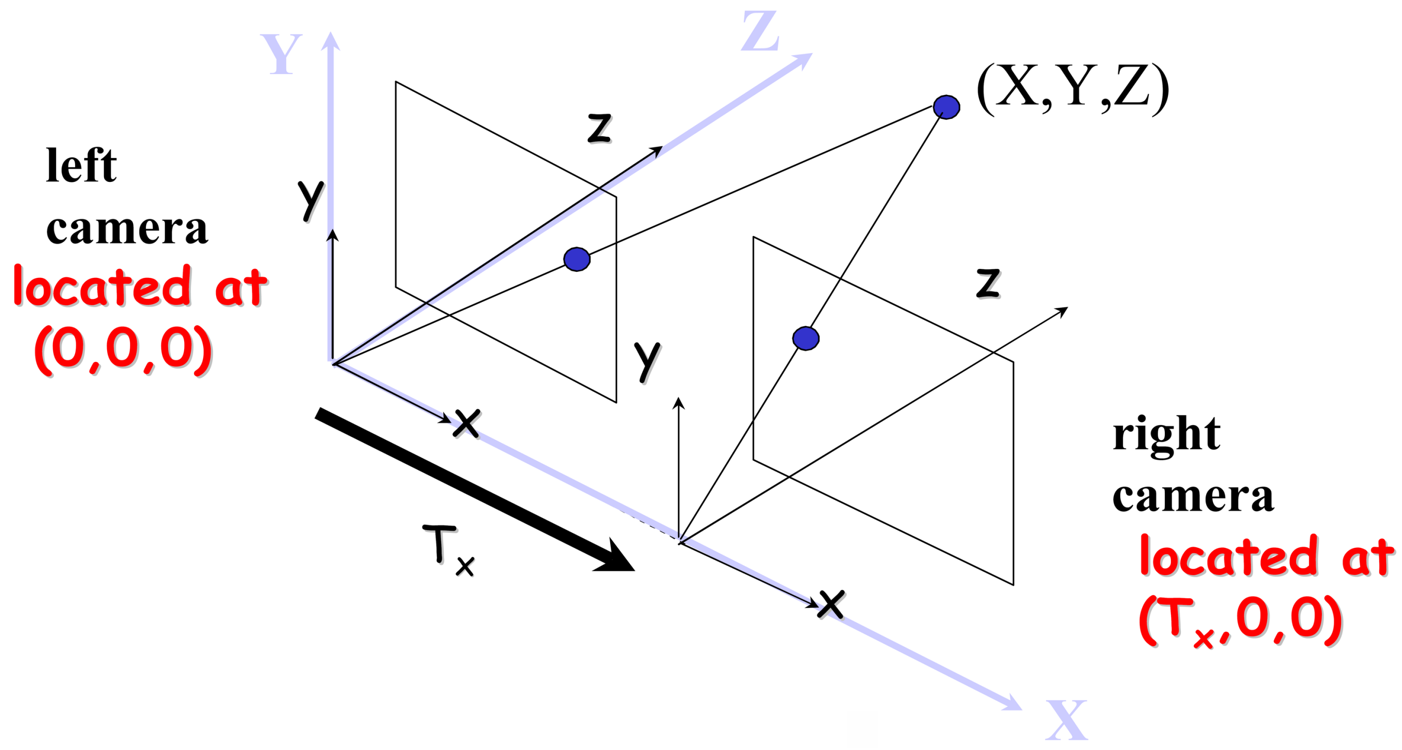
$$\det \mathbf{F} = 0$$

Special Case: A Stereo System



- The right camera is shifted by T_x (the stereo baseline)
- The camera intrinsics are the same

Special Case: A Stereo System



- Left camera

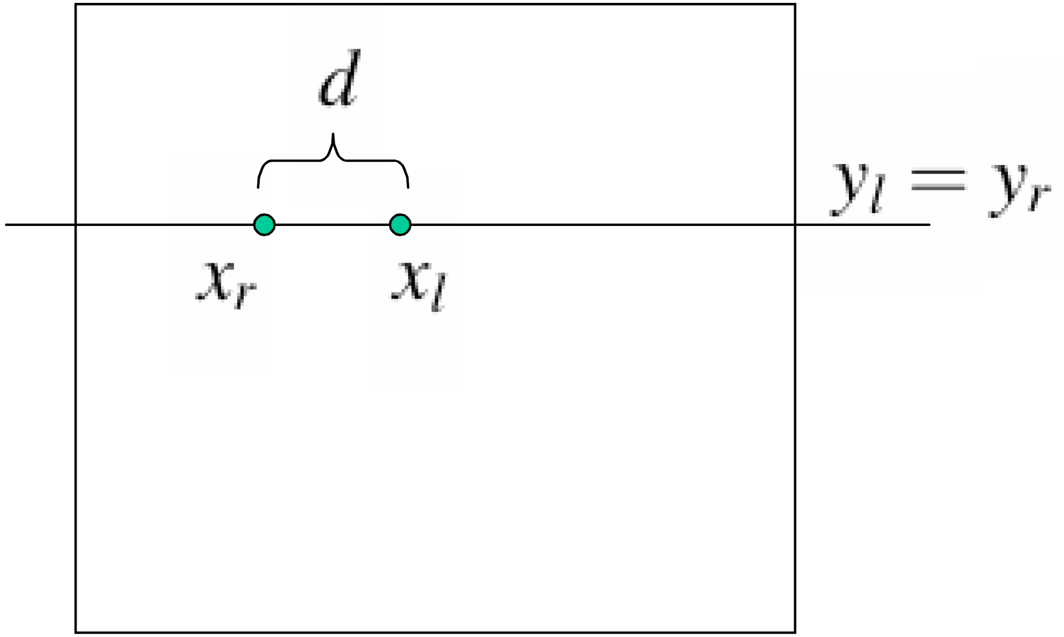
$$x_l = f\frac{X}{Z} + p_x \qquad y_l = f\frac{Y}{Z} + p_y$$

- Right camera

$$x_r = f\frac{X - T_x}{Z} + p_x$$

$$y_r = f\frac{Y}{Z} + p_y$$

Stereo Disparity



- Disparity

$$
\begin{aligned}
d &= x_l - x_r \\
&= \left(f\frac{X}{Z} + p_x\right) - \left(f\frac{X - T_x}{Z} + p_x\right) \\
&= f\frac{T_x}{Z}
\end{aligned}
$$

- Depth

$$Z = f\frac{T_x}{d}$$

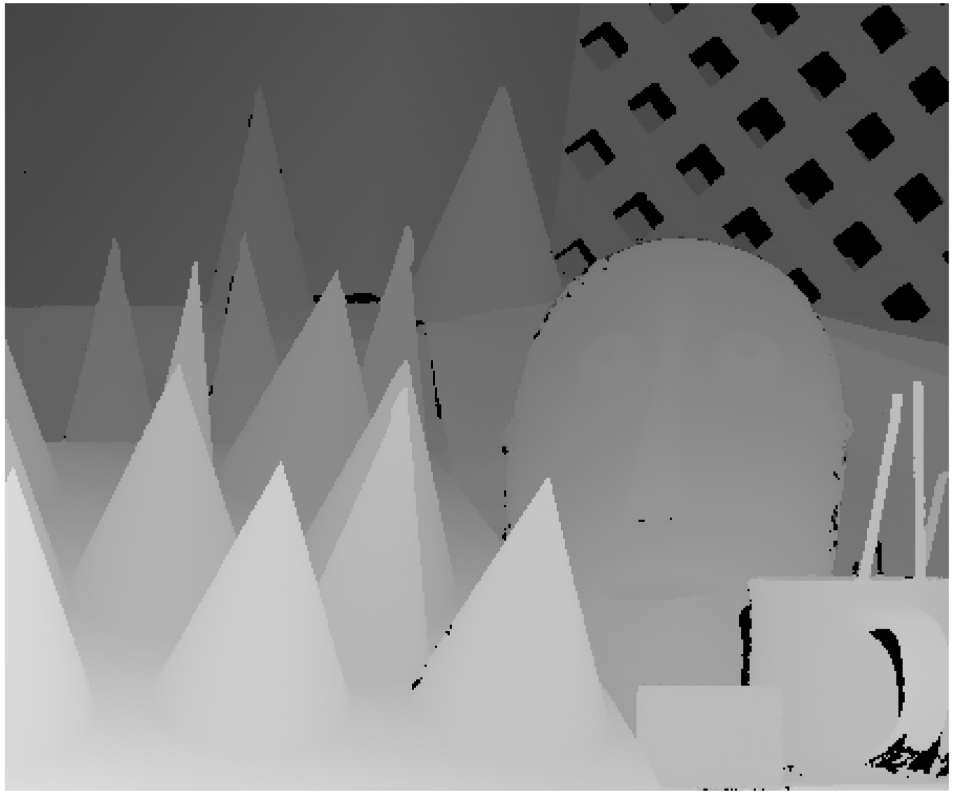
Baseline (T_x)

Disparity (d)

Recall motion parallax: near objects move faster (large disparity)

Stereo Example

Disparity values (0-64)

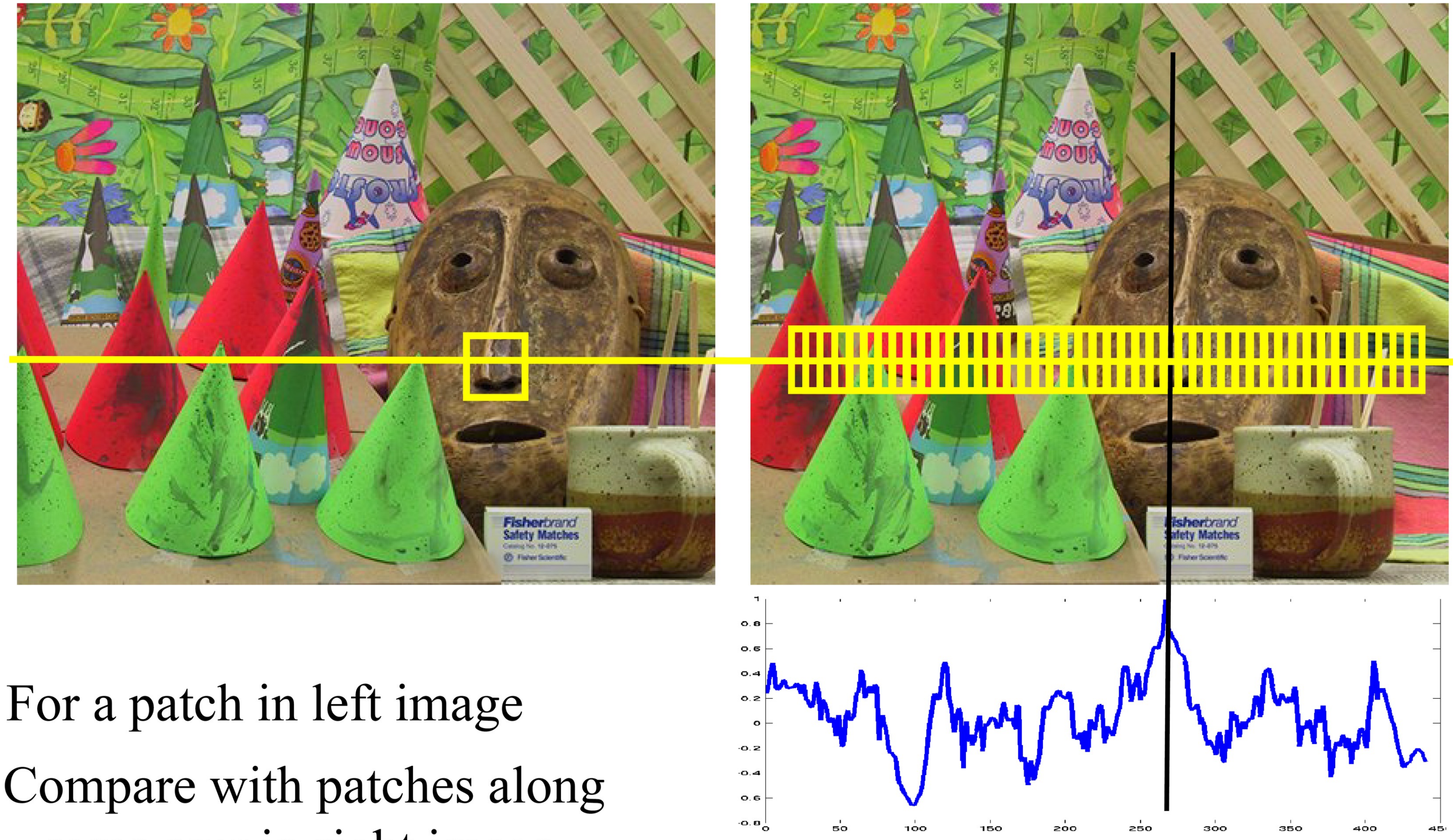


$$d = f\frac{T_x}{Z}$$

Note how disparity is larger (brighter) for closer surfaces.

Computing Disparity

Left Image Right Image



For a patch in left image

Compare with patches along same row in right image

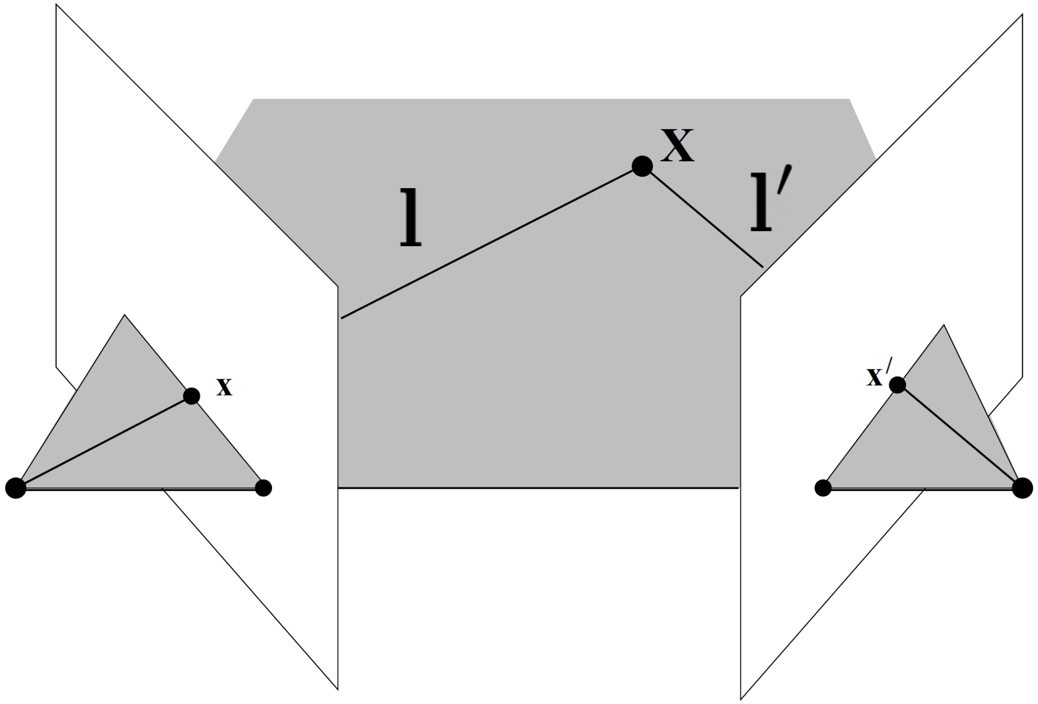
Match Score Values

- Eipipolar lines are horizontal lines in stereo
- For general cases, we can find correspondences on eipipolar lines
- Depth from disparity

$$Z = f\frac{T_x}{d}$$

Triangulation

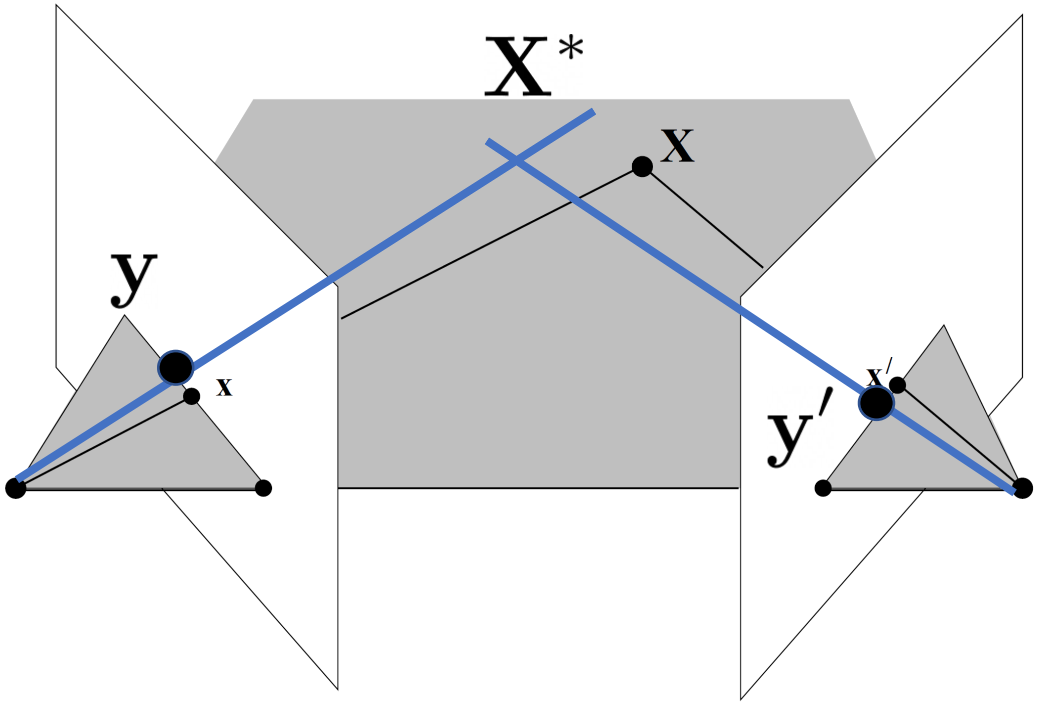
- Compute the 3D point given image correspondences



Intersection of two backprojected lines

$$\mathbf{X} = \mathbf{l} \times \mathbf{l}'$$

Triangulation



- In practice, we find the correspondences
  $\mathbf{y} \;\; \mathbf{y}'$
- The backprojected lines may not intersect
- Find X* that minimizes

$$d(\mathbf{y}, P\mathbf{X}^*) + d(\mathbf{y}', P'\mathbf{X}^*)$$

Projection matrix

Summary

- Depth perception
  - Monocular cues
  - Stereo cues
- Computational models for stereo vision
  - Epipolar geometry
  - Stereo Systems
  - Triangulation

Further Reading

- Section 6.1, Virtual Reality, Steven LaValle
- Multiview Geometry in Computer Vision, Richard Hartley and Andrew Zisserman, Chapter 9, Epipolar Geometry and Fundamental Matrix
- Stanford CS231A: Computer Vision, From 3D Reconstruction to Recognition, Lecture 5
  https://web.stanford.edu/class/cs231a/syllabus.html